Chapter 10: Square Root Functions and Geometry

10.1 Graphing Square Root Functions
10.2 Solving Square Root Equations
10.3 The Pythagorean Theorem
10.4 Using the Pythagorean Theorem

Let’s figure out how we can measure the height of the giant hyena standing right behind you.

I’m pretty sure that Pythagoras was a Greek.

I said ‘Greek’, not ‘Geek’.

The giant what?

Let’s figure out how we can measure the height of the giant hyena standing right behind you.

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What You Learned Before

○ Evaluating an Expression Involving a Square Root (8.EE.2)

Example 1 Evaluate $-4(\sqrt{121} - 16)$.

$-4(\sqrt{121} - 16) = -4(11 - 16)$ Evaluate the square root.

$= -4(-5)$ Subtract.

$= 20$ Multiply.

Try It Yourself
Evaluate the expression.

1. $7\sqrt{25} + 10$
2. $-8 - \frac{64}{16}$
3. $-2(3\sqrt{4} + 13)$

○ Factoring $x^2 + bx + c$ (A.SSE.3a)

Example 2 Factor $x^2 - 3x - 28$.

Notice that $b = -3$ and $c = -28$. Because $c$ is negative, the factors $p$ and $q$ must have different signs so that $pq$ is negative.

Find two integer factors of $-28$ whose sum is $-3$.

<table>
<thead>
<tr>
<th>Factors of $-28$</th>
<th>$-28$, 1</th>
<th>$-1$, 28</th>
<th>$-14$, 2</th>
<th>$-2$, 14</th>
<th>$-7$, 4</th>
<th>$-4$, 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum of Factors</td>
<td>$-27$</td>
<td>$27$</td>
<td>$-12$</td>
<td>$12$</td>
<td>$-3$</td>
<td>$3$</td>
</tr>
</tbody>
</table>

The values of $p$ and $q$ are $-7$ and $4$.

So, $x^2 - 3x - 28 = (x - 7)(x + 4)$.

Try It Yourself
Factor the polynomial.

4. $y^2 + 12y + 27$  5. $n^2 - 11n + 10$
6. $w^2 - 2w - 48$  7. $z^2 + 25z + 100$
Essential Question: How can you sketch the graph of a square root function?

### ACTIVITY: Graphing Square Root Functions

**Work with a partner.**

- Make a table of values for the function.
- Use the table to sketch the graph of the function.
- Describe the domain of the function.
- Describe the range of the function.

#### a. \( y = \sqrt{x} \)

#### b. \( y = \sqrt{x} + 2 \)

#### c. \( y = \sqrt{x + 1} \)

#### d. \( y = -\sqrt{x} \)

### COMMON CORE

**Square Root Functions**

In this lesson, you will

- graph square root functions.
- compare graphs of square root functions.

Learning Standards

F.IF.4
F.IF.7b

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ACTIVITY: Writing Square Root Functions

Work with a partner. Write a square root function, \( y = f(x) \), that has the given values. Then use the function to complete the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>0</td>
</tr>
<tr>
<td>-3</td>
<td>1</td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>1</td>
</tr>
<tr>
<td>-3</td>
<td>2</td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

ACTIVITY: Writing a Square Root Function

Work with a partner. Write a square root function, \( y = f(x) \), that has the given points on its graph. Explain how you found your function.

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you sketch the graph of a square root function? Summarize a procedure for sketching the graph. Then use your procedure to sketch the graph of each function.

a. \( y = 2\sqrt{x} \)

b. \( y = \sqrt{x} - 1 \)

c. \( y = \sqrt{x} - 1 \)

d. \( y = -2\sqrt{x} \)

Practice

Use what you learned about the graphs of square root functions to complete Exercises 3–8 on page 506.
**Key Idea**

**Square Root Function**

A square root function is a function that contains a square root with the independent variable in the radicand. The most basic square root function is \( y = \sqrt{x} \).

The value of the radicand in the square root function cannot be negative. So, the domain of a square root function includes \( x \)-values for which the radicand is greater than or equal to 0.

**EXAMPLE 1 Finding the Domain of a Square Root Function**

Find the domain of \( y = 3\sqrt{x - 5} \).

The radicand cannot be negative. So, \( x - 5 \) is greater than or equal to 0.

\[
\begin{align*}
x - 5 &\geq 0 \\
&\text{Write an inequality for the domain.} \\
x &\geq 5 \\
&\text{Add 5 to each side.}
\end{align*}
\]

\( \therefore \) The domain is the set of real numbers greater than or equal to 5.

**On Your Own**

Find the domain of the function.

1. \( y = 10\sqrt{x} \)
2. \( y = \sqrt{x} + 7 \)
3. \( y = \sqrt{-x + 1} \)

**EXAMPLE 2 Comparing Graphs of Square Root Functions**

Graph \( y = \sqrt{x} + 3 \). Describe the domain and range. Compare the graph to the graph of \( y = \sqrt{x} \).

**Step 1:** Make a table of values.

\[
\begin{array}{c|cccccc}
\hline
x & 0 & 1 & 4 & 9 & 16 \\
\hline
y & 3 & 4 & 5 & 6 & 7 \\
\hline
\end{array}
\]

**Step 2:** Plot the ordered pairs.

**Step 3:** Draw a smooth curve through the points.

\( \therefore \) From the graph, you can see that the domain is \( x \geq 0 \) and the range is \( y \geq 3 \). The graph of \( y = \sqrt{x} + 3 \) is a translation 3 units up of the graph of \( y = \sqrt{x} \).
EXAMPLE 3 Comparing Graphs of Square Root Functions

Graph \( y = -\sqrt{x - 2} \). Describe the domain and range. Compare the graph to the graph of \( y = \sqrt{x} \).

Step 1: Make a table of values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0</td>
<td>-1</td>
<td>-1.4</td>
<td>-1.7</td>
<td>-2</td>
</tr>
</tbody>
</table>

Step 2: Plot the ordered pairs.
Step 3: Draw a smooth curve through the points.

\(\because\) From the graph, you can see that the domain is \( x \geq 2 \) and the range is \( y \leq 0 \). The graph of \( y = -\sqrt{x - 2} \) is a reflection of the graph of \( y = \sqrt{x} \) in the \( x \)-axis and then a translation 2 units to the right.

Now You’re Ready

Exercises 16–21

On Your Own

Graph the function. Describe the domain and range. Compare the graph to the graph of \( y = \sqrt{x} \).

4. \( y = \sqrt{x} - 4 \)
5. \( y = \sqrt{x} + 5 \)
6. \( y = -\sqrt{x + 1} + 2 \)

EXAMPLE 4 Real-Life Application

The velocity \( y \) (in meters per second) of a tsunami can be modeled by the function \( y = \sqrt{9.8x} \), where \( x \) is the water depth (in meters). Use a graphing calculator to graph the function. At what depth does the velocity of the tsunami exceed 200 meters per second?

Step 1: Enter the function \( y = \sqrt{9.8x} \) into your calculator and graph it. Because the radicand cannot be negative, use only nonnegative values of \( x \).
Step 2: Use the trace feature to find where the value of \( y \) is about 200.

\(\because\) The velocity exceeds 200 meters per second at a depth of about 4100 meters.

On Your Own

7. Find the domain and range of the function in Example 4.
8. WHAT IF? In Example 4, at what depth does the velocity of the tsunami exceed 100 meters per second?
10.1 Exercises

Vocabulary and Concept Check

1. **VOCABULARY** Is \( y = 2x\sqrt{5} \) a square root function? Explain.

2. **REASONING** How do you find the domain of a square root function?

Practice and Problem Solving

Match the function with its graph.

3. \( y = 8\sqrt{x} \)

4. \( y = \frac{5}{4}\sqrt{x} \)

5. \( y = -4\sqrt{x} \)

Graph the function. Describe the domain.

6. \( y = 3\sqrt{x} \)

7. \( y = 7\sqrt{x} \)

8. \( y = -0.5\sqrt{x} \)

Find the domain of the function.

9. \( y = 5\sqrt{x} \)

10. \( y = \sqrt{x} + 1 \)

11. \( y = \sqrt{x} - 2 \)

12. \( y = \sqrt{-x} - 1 \)

13. \( y = 2\sqrt{x} + 4 \)

14. \( y = \frac{1}{2}\sqrt{-x} + 2 \)

15. **FIRE** The nozzle pressure of a fire hose allows firefighters to control the amount of water they spray on a fire. The flow rate \( f \) (in gallons per minute) can be modeled by the function \( f = 120\sqrt{p} \), where \( p \) is the nozzle pressure (in pounds per square inch).

   a. Use a graphing calculator to graph the function.

   b. Use the **trace** feature to approximate the nozzle pressure that results in a flow rate of 300 gallons per minute.
Graph the function. Describe the domain and range. Compare the graph to the graph of \( y = \sqrt{x} \).

16. \( y = \sqrt{x} - 2 \)
17. \( y = \sqrt{x} + 4 \)
18. \( y = \sqrt{x} + 4 \)
19. \( y = \sqrt{x} + 2 - 2 \)
20. \( y = -\sqrt{x} - 3 \)
21. \( y = -\sqrt{x} - 1 + 3 \)

22. **ERROR ANALYSIS** Describe and correct the error in graphing the function \( y = \sqrt{x} + 1 \).

23. **OPEN-ENDED** Consider the graph of \( y = \sqrt{x} \).
   a. Write a function that is a vertical translation of the graph of \( y = \sqrt{x} \).
   b. Write a function that is a reflection of the graph of \( y = \sqrt{x} \).

24. **REASONING** Can the domain of a square root function include negative numbers? Can the range include negative numbers? Explain your reasoning.

25. **GEOMETRY** The radius of a circle is given by \( r = \frac{\sqrt{A}}{\pi} \), where \( A \) is the area of the circle.
   a. Find the domain of the function. Use a graphing calculator to graph the function.
   b. Use the *trace* feature to approximate the area of a circle with a radius of 3 inches.

26. **PROBLEM SOLVING** The speed \( S \) (in miles per hour) of a van before it skids to a stop can be modeled by the equation \( S = \sqrt{\frac{30d}{f}} \), where \( d \) is the length (in feet) of the skid marks and \( f \) is the drag factor of the road surface. Suppose the drag factor is 0.75 and the speed of the van was 40 miles per hour. Is the length of the skid marks more than 65 feet long? Explain your reasoning.

27. **Precision** Compare the graphs of the functions \( f(x) = \sqrt{x} \) and \( g(x) = 3\sqrt{x} \).

---

**Fair Game Review** What you learned in previous grades & lessons

Solve the equation. *(Section 7.5)*

28. \( x(x - 8) = 0 \)
29. \( (x + 3)^2 = 0 \)
30. \( (x + 2)(x - 3) = 0 \)

31. **MULTIPLE CHOICE** What are the next three terms of the geometric sequence 240, 120, 60, 30, . . .? *(Section 6.7)*
   A. 20, 10, 5
   B. 15, 7.5, 3.75
   C. 20, 10, 0
   D. 15, 10, 5
In Section 6.1, you used properties to simplify radical expressions. A radical expression is in **simplest form** when the following are true.

- No radicands have perfect square factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

When a radicand in the denominator of a fraction is not a perfect square, multiply the fraction by an appropriate form of 1 to eliminate the radical from the denominator. This process is called **rationalizing the denominator**.

### EXAMPLE 1 Simplifying a Radical Expression

Simplify \( \frac{1}{\sqrt{3}} \).

\[
\frac{1}{\sqrt{3}} = \frac{\sqrt{1}}{\sqrt{3}} \quad \text{Quotient Property of Square Roots}
\]

\[
= \frac{\sqrt{1} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} \quad \text{Multiply by} \ \frac{\sqrt{3}}{\sqrt{3}}
\]

\[
= \frac{\sqrt{1} \cdot 3}{\sqrt{3} \cdot 3} \quad \text{Product Property of Square Roots}
\]

\[
= \frac{\sqrt{3}}{\sqrt{9}} \quad \text{Simplify.}
\]

\[
= \frac{\sqrt{3}}{3} \quad \text{Evaluate the square root.}
\]

### Practice

Simplify the expression.

1. \( \frac{1}{\sqrt{10}} \)
2. \( \frac{\sqrt{2}}{\sqrt{7}} \)
3. \( \sqrt{\frac{9}{2}} \)
4. \( \sqrt{\frac{10}{21}} \)
5. \( \frac{\sqrt{18}}{\sqrt{18}} \)
6. \( \sqrt{\frac{40}{48}} \)
7. \( \frac{4}{\sqrt{5}} + \frac{1}{\sqrt{5}} \)
8. \( \sqrt{3} - \frac{2}{\sqrt{12}} \)
9. \( \sqrt{\frac{16}{15} - \frac{1}{3}} \)

10. **REASONING** Explain why for any number \( a \), \( \sqrt{a^2} = |a| \). Use this rule to simplify \( \sqrt{\frac{x^2}{2}} \).
The binomials \(a\sqrt{b} + c\sqrt{d}\) and \(a\sqrt{b} - c\sqrt{d}\) are called **conjugates**. You can use conjugates to simplify radical expressions that involve a sum or difference of radicals in the denominator.

**Example 2** Simplifying a Radical Expression

Simplify \(\frac{1}{3 + \sqrt{5}}\).

\[
\frac{1}{3 + \sqrt{5}} = \frac{1}{3 + \sqrt{5}} \cdot \frac{3 - \sqrt{5}}{3 - \sqrt{5}} = \frac{1(3 - \sqrt{5})}{3^2 - (\sqrt{5})^2} = \frac{3 - \sqrt{5}}{4}
\]

The conjugate of \(3 + \sqrt{5}\) is \(3 - \sqrt{5}\).

**Sum and Difference Pattern**

Simplify.

**Example 3** Real-Life Application

The distance \(d\) (in miles) that you can see to the horizon with your eye level \(h\) feet above the water is given by \(d = \sqrt{\frac{3h}{2}}\). How far can you see when your eye level is 5 feet above the water?

\[
d = \sqrt{\frac{3(5)}{2}} \quad \text{Substitute 5 for } h.
\]

\[
= \frac{\sqrt{15}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \quad \text{Multiply by } \frac{\sqrt{2}}{\sqrt{2}}.
\]

\[
= \frac{\sqrt{30}}{2} \quad \text{Simplify.}
\]

You can see \(\frac{\sqrt{30}}{2}\), or about 2.74 miles.

**Practice**

Simplify the expression.

11. \(\frac{6}{1 + \sqrt{3}}\)  

12. \(\frac{5}{\sqrt{3} - 2}\)  

13. \(\frac{10}{\sqrt{2} + \sqrt{7}}\)

14. **WHAT IF?** In Example 3, how far can you see when your eye level is 35 feet above the water?
10.2 Solving Square Root Equations

Essential Question How can you solve an equation that contains square roots?

1 ACTIVITY: Analyzing a Free-Falling Object

Work with a partner. The table shows the time \( t \) (in seconds) that it takes a free-falling object (with no air resistance) to fall \( d \) feet.

a. Sketch the graph of \( t \) as a function of \( d \).

b. Use your graph to estimate the time it takes for a free-falling object to fall 240 feet.

c. The relationship between \( d \) and \( t \) is given by the function

\[
t = \sqrt{\frac{d}{16}}.\]

Use this function to check the estimate you obtained from the graph.

d. Consider a free-falling object that takes 5 seconds to hit the ground. How far did it fall? Explain your reasoning.

<table>
<thead>
<tr>
<th>( d ) feet</th>
<th>( t ) seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>32</td>
<td>1.41</td>
</tr>
<tr>
<td>64</td>
<td>2.00</td>
</tr>
<tr>
<td>96</td>
<td>2.45</td>
</tr>
<tr>
<td>128</td>
<td>2.83</td>
</tr>
<tr>
<td>160</td>
<td>3.16</td>
</tr>
<tr>
<td>192</td>
<td>3.46</td>
</tr>
<tr>
<td>224</td>
<td>3.74</td>
</tr>
<tr>
<td>256</td>
<td>4.00</td>
</tr>
<tr>
<td>288</td>
<td>4.24</td>
</tr>
<tr>
<td>320</td>
<td>4.47</td>
</tr>
</tbody>
</table>

Common Core Radical Functions

In this lesson, you will
- solve square root equations, including those with square roots on both sides.
- identify extraneous solutions.

Applying Standard N.RN.2
2. **ACTIVITY: Solving a Square Root Equation**

Work with a partner. Sketch the graph of each function. Then find the value of $x$ such that $f(x) = 2$. Explain your reasoning.

a. $f(x) = \sqrt{x - 2}$

b. $f(x) = \sqrt{x} - 1$

3. **ACTIVITY: Solving a Square Root Equation**

Work with a partner. The speed $s$ (in feet per second) of the free-falling object in Activity 1 is given by the function

$$s = \sqrt{64d}.$$  

Find the distance traveled for each speed.

a. $s = 8$ ft/sec  

b. $s = 16$ ft/sec  

c. $s = 24$ ft/sec

4. **IN YOUR OWN WORDS** How can you solve an equation that contains square roots? Summarize a procedure for solving a square root equation. Then use your procedure to solve each equation.

a. $\sqrt{x} + 2 = 3$  

b. $4 - \sqrt{x} = 1$  

c. $5 = \sqrt{x} + 20$  

d. $-3 = -2\sqrt{x}$

**Practice**

Use what you learned about solving square root equations to complete Exercises 3–5 on page 515.
A **square root equation** is an equation that contains a square root with a variable in the radicand. To solve a square root equation, use properties of equality to isolate the square root by itself on one side of the equation, then use the following property.

### Key Idea

**Squaring Each Side of an Equation**

**Words** If two expressions are equal, then their squares are also equal.

**Algebra** If \( a = b \), then \( a^2 = b^2 \).

### Example 1

**Solving Square Root Equations**

a. Solve \( \sqrt{x} + 5 = 13 \).

\[
\begin{align*}
\sqrt{x} + 5 &= 13 \\
\sqrt{x} &= 8 \\
(\sqrt{x})^2 &= 8^2 \\
x &= 64 \\
8 + 5 &= 13 \\
13 &= 13 \quad \checkmark
\end{align*}
\]

\( \therefore \) The solution is \( x = 64 \).

b. Solve \( 3 - \sqrt{x} = 0 \).

\[
\begin{align*}
3 - \sqrt{x} &= 0 \\
3 &= \sqrt{x} \\
3^2 &= (\sqrt{x})^2 \\
9 &= x \\
3^2 &= (\sqrt{x})^2 \\
9 &= x \\
\therefore \) The solution is \( x = 9 \).
\]

### On Your Own

Solve the equation. Check your solution.

1. \( \sqrt{x} = 6 \)
2. \( \sqrt{x} - 7 = 3 \)
3. \( \sqrt{x} + 15 = 22 \)
4. \( 1 - \sqrt{x} = -2 \)
EXAMPLE 2  Solving a Square Root Equation

Solve $4\sqrt{x + 2} + 3 = 19$.

$4\sqrt{x + 2} + 3 = 19$  
Write the equation.

$4\sqrt{x + 2} = 16$  
Subtract 3 from each side.

$\sqrt{x + 2} = 4$  
Divide each side by 4.

$(\sqrt{x + 2})^2 = 4^2$  
Square each side of the equation.

$x + 2 = 16$  
Simplify.

$x = 14$  
Subtract 2 from each side.

The solution is $x = 14$.

**On Your Own**

Solve the equation. Check your solution.

5. $\sqrt{x + 4} + 7 = 11$  
6. $8\sqrt{x - 1} = 24$  
7. $15 = 6 + \sqrt{3x - 9}$

EXAMPLE 3  Solving an Equation with Square Roots on Both Sides

Solve $\sqrt{2x - 1} = \sqrt{x + 4}$.

$\sqrt{2x - 1} = \sqrt{x + 4}$  
Write the equation.

$(\sqrt{2x - 1})^2 = (\sqrt{x + 4})^2$  
Square each side of the equation.

$2x - 1 = x + 4$  
Simplify.

$x - 1 = 4$  
Subtract $x$ from each side.

$x = 5$  
Add 1 to each side.

The solution is $x = 5$.

**On Your Own**

Solve the equation. Check your solution.

8. $\sqrt{3x + 1} = \sqrt{4x - 7}$  
9. $\sqrt{x} = \sqrt{5x - 1}$

Squaring each side of an equation can sometimes introduce a solution that is *not* a solution of the original equation. This solution is called an *extraneous solution*. Be sure to always substitute your solutions into the original equation to check for extraneous solutions.
EXAMPLE 4 Identifying an Extraneous Solution

\[
x = \sqrt{x + 6}
\]

Original equation

\[
x^2 = (\sqrt{x + 6})^2
\]

Square each side of the equation.

\[
x^2 = x + 6
\]

Simplify.

\[
x^2 - x - 6 = 0
\]

Subtract x and 6 from each side.

\[
(x - 3)(x + 2) = 0
\]

Factor.

\[
x - 3 = 0 \quad \text{or} \quad x + 2 = 0
\]

Use Zero-Product Property.

\[
x = 3 \quad \text{or} \quad x = -2
\]

Solve for x.

Check

\[
3 \not= \sqrt{3 + 6}
\]

Substitute for x.

\[
3 \not= \sqrt{9}
\]

Simplify.

\[
3 = 3 \checkmark
\]

Because \(x = -2\) does not check in the original equation, it is an extraneous solution. The only solution is \(x = 3\).

EXAMPLE 5 Real-Life Application

The period \(P\) (in seconds) of a pendulum is given by the function \(P = 2\pi \sqrt{\frac{L}{32}}\), where \(L\) is the pendulum length (in feet). What is the length of a pendulum that has a period of 2 seconds?

\[
2 = 2\pi \sqrt{\frac{L}{32}}
\]

Substitute 2 for \(P\) in the function.

\[
\frac{1}{\pi} = \sqrt{\frac{L}{32}}
\]

Divide each side by \(2\pi\) and simplify.

\[
\frac{1}{\pi^2} = \frac{L}{32}
\]

Square each side and simplify.

\[
\frac{32}{\pi^2} = L
\]

Multiply both sides by 32.

\[
3.2 \approx L
\]

Use a calculator.

\[
\therefore \quad \text{The length of the pendulum is about 3.2 feet.}
\]

On Your Own

10. Solve \(\sqrt{x} - 1 = x - 3\). Check your solution.

11. WHAT IF? In Example 5, what is the length of a pendulum that has a period of 4 seconds? Is your result twice the length in Example 5? Explain.
1. **VOCABULARY** Is \( \sqrt{\frac{3}{4}} = 4 \) a square root equation? Explain your reasoning.

2. **WRITING** Why should you check every solution of a square root equation?

### Practice and Problem Solving

Sketch the graph of the function. Then find the value of \( x \) such that \( f(x) = 3 \).

3. \( f(x) = \sqrt{x} + 1 \)
4. \( f(x) = \sqrt{x} - 3 \)
5. \( f(x) = \sqrt{x} + 1 - 2 \)

Solve the equation. Check your solution.

6. \( \sqrt{x} = 9 \)
7. \( 7 = \sqrt{x} - 5 \)
8. \( \sqrt{x} + 6 = 10 \)
9. \( \sqrt{x} + 12 = 23 \)
10. \( 4 - \sqrt{x} = 4 \)
11. \( 8 = 7 - \sqrt{x} \)

12. **ERROR ANALYSIS** Describe and correct the error in solving the equation.

\[ 4 - \sqrt{x} = 2 \]
\[ 7 = -\sqrt{x} = -2 \]
\[ -x = 4 \]
\[ x = -4 \]

Solve the equation. Check your solution.

13. \( \sqrt{x} - 3 + 5 = 9 \)
14. \( 2\sqrt{x} + 4 = 16 \)
15. \( 25 = 7 + 3\sqrt{x} - 9 \)
16. \( \sqrt{\frac{x}{2}} + 14 = 18 \)
17. \( -1 = \sqrt{5x + 1} - 7 \)
18. \( 12 = 19 - \sqrt{3x - 11} \)

19. **CUBE** The formula \( s = \sqrt[3]{A} \) gives the edge length \( s \) of a cube with a surface area of \( A \). What is the surface area of a cube with an edge length of 4 inches?

20. **BASE JUMPING** The Cave of Swallows is a natural open-air pit cave in the state of San Luis Potosi, Mexico. The 1220-foot deep cave is a popular destination for BASE jumpers. The formula \( t = \sqrt[2]{\frac{d}{16}} \) gives the distance \( d \) (in feet) a BASE jumper free falls in \( t \) seconds. How far does the BASE jumper fall in 3 seconds?

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Section 10.2 Solving Square Root Equations 515
21. **WRITING** Explain how you would solve \( \sqrt{m + 4} - \sqrt{3m} = 0 \).

Solve the equation. Check your solution.

22. \( \sqrt{2x - 9} = \sqrt{x} \)

23. \( \sqrt{x + 1} = \sqrt{4x - 8} \)

24. \( \sqrt{3x + 1} = \sqrt{7x - 19} \)

25. \( \sqrt{8x - 7} = \sqrt{6x + 7} \)

26. \( \sqrt{2x + 1} - \sqrt{4x} = 0 \)

27. \( \sqrt{5x - 7} = \sqrt{8x - 2} = 0 \)

Find the value of \( x \).

28. Perimeter = 28 cm

29. Area = \( \sqrt{5x - 4} \) ft

30. **OPEN-ENDED** Write a square root equation of the form \( \sqrt{ax + b} = c \) that has a solution of 9.

Solve the equation. Check your solution.

31. \( x = \sqrt{5x - 4} \)

32. \( \sqrt{9x - 14} = x \)

33. \( \sqrt{3x + 10} = x \)

34. \( 2x = \sqrt{6 - 10x} \)

35. \( x - 1 = \sqrt{3 - x} \)

36. \( \sqrt{-4x - 19} = x + 4 \)

37. **ERROR ANALYSIS** Describe and correct the error in solving the equation.

38. **REASONING** Explain how to use mental math to find the solution of \( \sqrt{2x + 5} = 1 \).

Determine whether the statement is **true or false**.

39. If \( \sqrt{a} = b \), then \( (\sqrt{a})^2 = b^2 \).

40. If \( \sqrt{a} = \sqrt{b} \), then \( a = b \).

41. If \( a^2 = b^2 \), then \( a = b \).

42. If \( a^2 = \sqrt{b} \), then \( a^4 = (\sqrt{b})^2 \).

43. **ELECTRICITY** The formula \( V = \sqrt{PR} \) relates the voltage \( V \) (in volts), power \( P \) (in watts), and resistance \( R \) (in ohms) of an electrical circuit. What is the resistance of a 1000-watt hair dryer on a 120-volt circuit?
44. **CHOOSE TOOLS** Consider the equation \( x + 2 = \sqrt{2x - 3} \).

a. Graph each side of the equation in the same coordinate plane. Solve the equation by finding points of intersection.

b. Solve the equation algebraically. How does your solution compare to the solution in part (a)?

c. Which method do you prefer? Explain your reasoning.

45. **TRAPEZE** The time \( t \) (in seconds) it takes a trapeze artist to swing back and forth is given by the function \( t = 2\pi \frac{r}{32} \), where \( r \) is the rope length (in feet). It takes 6 seconds to swing back and forth. How long is the rope? Use 3.14 for \( \pi \).

46. **GEOMETRY** The formula \( s = \sqrt{r^2 + h^2} \) gives the slant height \( s \) of a cone, where \( r \) is the radius of the base, and \( h \) is the height. The slant heights of the two cones are equal. Find the radius of each cone.

47. **CRITICAL THINKING** How is squaring \( \sqrt{x} + 2 \) different than squaring \( \sqrt{x} + 2^2 \)?

Solve the equation. Check your solution.

48. \( \sqrt{x} + 15 = \sqrt{x} + \sqrt{5} \)

49. \( 2 - \sqrt{x} + 1 = \sqrt{x} + 2 \)

50. **Modeling** The formula \( h = \sqrt{2A - b_2h} \) gives the height \( h \) of the speaker box, where \( A \) is the area of one trapezoidal side, and \( b_2 \) is the length of base 2.

a. Given that \( A = 168 \) square inches and \( b_2 = 16 \) inches, find \( h \).

b. What is the length of \( b_1 \) (base 1)?

c. Speakers work best when the volume of the speaker box is \( \pm 10\% \) of the manufacturer's recommendation. Find the range of the widths \( w \) when the manufacturer recommends a volume of 1.5 cubic feet.

---

**Fair Game Review** What you learned in previous grades & lessons

Two angle measures of a triangle are given. Find the measure of the missing angle. (Skills Review Handbook)

51. \( 40^\circ, 48^\circ \)

52. \( 45^\circ, 55^\circ \)

53. \( 36^\circ, 54^\circ \)

54. **MULTIPLE CHOICE** Which function is represented by the ordered pairs \((-1, 0.5), (0, 1), (1, 2), (2, 4), \) and \((3, 8)\)? (Section 8.5)

A \( y = 0.5x^2 \)  
B \( y = 2^x \)  
C \( y = 2x^2 \)  
D \( y = 2x \)
You can use a **word magnet** to organize information associated with a vocabulary word. Here is an example of a word magnet for square root functions.

**Definition:** A function that contains a square root with the independent variable in the radicand.

**Examples:**
- \( y = \sqrt{x} + 3 \)
- \( y = \sqrt{x - 1} \)
- \( y = \sqrt{x + 5} - 4 \)

**Sample Graph:**

### Domain:
The value of the radicand cannot be negative. So, the domain is limited to \( x \)-values for which the radicand is greater than or equal to 0.

### Graph:
Make a table of values. Plot the ordered pairs. Draw a smooth curve through the points. Find the domain and range.

### Compare:
When graphing a square root function \( f(x) \):
- \( f(x) + k \) is a vertical translation of \( f(x) \).
- \( f(x + h) \) is a horizontal translation of \( f(x) \).
- \(-f(x)\) is a reflection of \( f(x) \) in the \( x \)-axis.

---

### On Your Own

Make word magnets to help you study these topics.

1. rationalizing the denominator
2. solving a square root equation
3. extraneous solution

After you complete this chapter, make word magnets for the following topics.

4. Pythagorean Theorem
5. converse of the Pythagorean Theorem
6. distance formula
10.1–10.2 Quiz

Find the domain of the function.  (Section 10.1)
1. \( y = 15\sqrt{x} \)  
2. \( y = \sqrt{x} - 3 \)  
3. \( y = \sqrt{3 - x} \)

Graph the function. Describe the domain and range. Compare the graph to the graph of \( y = \sqrt{x} \).  (Section 10.1)
4. \( y = \sqrt{x} + 5 \)  
5. \( y = \sqrt{x} - 4 \)  
6. \( y = -\sqrt{x} - 2 + 1 \)

Simplify the expression.  (Section 10.1)
7. \( \sqrt{\frac{6}{42}} \)  
8. \( \frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}} \)  
9. \( \frac{7}{\sqrt{5} + 2} \)

Solve the equation.  (Section 10.2)
10. \( \sqrt{x} - 1 + 7 = 15 \)  
11. \( \sqrt{x} = \sqrt{6x - 20} \)  
12. \( x = \sqrt{21 - 4x} \)

Find the value of \( x \).  (Section 10.2)
13. Perimeter = 24 mi  
14. Area = \( 2\sqrt{4x - 7} \) m\(^2\)

15. **BRIDGE** The time \( t \) (in seconds) it takes an object to drop \( h \) feet is given by \( t = \frac{1}{4}\sqrt{h} \).  (Section 10.1)
   a. Graph the function. Describe the domain and range.
   b. It takes about 7.4 seconds for a stone dropped from the New River Gorge Bridge in West Virginia, to reach the water below. About how high is the bridge above the New River?

16. **SPEED OF SOUND** The speed of sound \( s \) (in meters per second) through air is given by \( s = 20\sqrt{T + 273} \), where \( T \) is the temperature in degrees Celsius.  (Section 10.2)
   a. What is the temperature when the speed of sound is 340 meters per second?
   b. How long does it take you to hear the wolf howl when the temperature is \(-17^\circ C\)?
The Pythagorean Theorem

Essential Question How are the lengths of the sides of a right triangle related?

Pythagoras was a Greek mathematician and philosopher who discovered one of the most famous rules in mathematics. In mathematics, a rule is called a theorem. So, the rule that Pythagoras discovered is called the Pythagorean Theorem.

Pythagoras (c. 570 B.C.–c. 490 B.C.)

ACTIVITY: Discovering the Pythagorean Theorem

Work with a partner.

a. On grid paper, draw any right triangle. Label the lengths of the two shorter sides (the legs) \(a\) and \(b\).

b. Label the length of the longest side (the hypotenuse) \(c\).

c. Draw squares along each of the three sides. Label the areas of the three squares \(a^2\), \(b^2\), and \(c^2\).

d. Cut out the three squares. Make eight copies of the right triangle and cut them out. Arrange the figures to form two identical larger squares.

e. What does this tell you about the relationship among \(a^2\), \(b^2\), and \(c^2\)?
2 **ACTIVITY: Finding the Length of the Hypotenuse**

Work with a partner. Use the result of Activity 1 to find the length of the hypotenuse of each right triangle.

a. 
\[
\text{6} \quad \text{8} \quad \sqrt{\text{c}}
\]

b. 
\[
\text{5} \quad \text{12} \quad \sqrt{\text{c}}
\]

c. 
\[
\frac{1}{3} \quad \frac{1}{4} \quad \sqrt{\text{c}}
\]

d. 
\[
\text{0.3} \quad \text{0.4} \quad \sqrt{\text{c}}
\]

3 **ACTIVITY: Finding the Length of a Leg**

Work with a partner. Use the result of Activity 1 to find the length of the leg of each right triangle.

a. 
\[
\text{a} \quad \text{15} \quad \sqrt{12}
\]

b. 
\[
2.4 \quad \text{4} \quad \sqrt{b}
\]

4. **IN YOUR OWN WORDS** How are the lengths of the sides of a right triangle related? Give an example using whole numbers.

**What Is Your Answer?**

Use what you learned about the Pythagorean Theorem to complete Exercises 3–5 on page 524.
### 10.3 Lesson

#### Key Vocabulary
- **Theorem**, p. 520
- **Legs**, p. 522
- **Hypotenuse**, p. 522
- **Pythagorean Theorem**, p. 522

#### Key Ideas

**Sides of a Right Triangle**
The sides of a right triangle have special names.

- **Legs** are the two sides that form the right angle.
- **Hypotenuse** is the side opposite the right angle.

#### Study Tip
In a right triangle, the legs are the shorter sides and the hypotenuse is always the longest side.

#### The Pythagorean Theorem

**Words**
In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

**Algebra**
\[ a^2 + b^2 = c^2 \]

#### Example 1: Finding the Length of a Hypotenuse

Find the length of the hypotenuse of the triangle.

\[
\begin{align*}
15^2 + 8^2 &= c^2 & \text{Write the Pythagorean Theorem.} \\
225 + 64 &= c^2 & \text{Substitute 15 for } a \text{ and 8 for } b. \\
289 &= c^2 & \text{Evaluate powers.} \\
\sqrt{289} &= \sqrt{c^2} & \text{Add.} \\
17 &= c & \text{Take positive square root of each side.}
\end{align*}
\]

The length of the hypotenuse is 17 feet.

#### On Your Own

Find the length of the hypotenuse of the triangle.

1. \[
\begin{align*}
7 \text{ cm} & \quad 24 \text{ cm} \\
c & \quad \text{c}
\end{align*}
\]

2. \[
\begin{align*}
7 \text{ in.} & \quad 10 \text{ in.} \\
c & \quad \text{c}
\end{align*}
\]
EXAMPLE 2 Finding the Length of a Leg

Find the missing length of the triangle.

\[ a^2 + b^2 = c^2 \]
Write the Pythagorean Theorem.

\[ 3.5^2 + b^2 = 6.5^2 \]
Substitute 3.5 for \( a \) and 6.5 for \( c \).

\[ 12.25 + b^2 = 42.25 \]
Evaluate powers.

\[ b^2 = 30 \]
Subtract 12.25 from each side.

\[ b = \sqrt{30} \]
Take positive square root of each side.

The length of the leg is \( \sqrt{30} \approx 5.5 \) kilometers.

EXAMPLE 3 Real-Life Application

Paintball Team A is located 70 feet north and 60 feet east of the base. Team B is located 30 feet north and 30 feet east of the base. How far apart are the teams?

Step 1: Draw the situation in a coordinate plane. Let the base be at the origin. From the descriptions, you can plot Team A at \((60, 70)\) and Team B at \((30, 30)\).

Step 2: Draw a right triangle with a hypotenuse that represents the distance between the teams. The lengths of the legs are 30 feet and 40 feet.

Step 3: Use the Pythagorean Theorem to find the length of the hypotenuse.

\[ a^2 + b^2 = c^2 \]
Write the Pythagorean Theorem.

\[ 30^2 + 40^2 = c^2 \]
Substitute 30 for \( a \) and 40 for \( b \).

\[ 900 + 1600 = c^2 \]
Evaluate powers.

\[ 2500 = c^2 \]
Add.

\[ 50 = c \]
Take positive square root of each side.

The teams are 50 feet apart.

On Your Own

Find the missing length of the triangle.

3.

4.

5. WHAT IF? In Example 3, Team B moves 10 feet to the west. How far apart are the teams to the nearest foot?
10.3 Exercises

Vocabulary and Concept Check

1. VOCABULARY You are given the lengths of the hypotenuse and one leg of a right triangle. Describe how you can find the length of the other leg.

2. DIFFERENT WORDS, SAME QUESTION Which is different? Find "both" answers.
   - Which side is a leg?
   - Which side is shortest?
   - Which side is longest?
   - Which side is part of a right angle?

Practice and Problem Solving

Find the missing length of the triangle.

3. \( a = 16 \text{ yd}, b = 20 \text{ yd} \)

4. \( a = 7 \text{ cm}, b = 9 \text{ cm} \)

5. \( b = 15 \text{ ft}, c = 32 \text{ ft} \)

6. \( a = 1 \text{ km}, c = 2.4 \text{ km} \)

7. \( b = \sqrt{61} \text{ in.}, c = 5 \text{ in.} \)

8. \( b = \frac{2}{5} \text{ mm}, c = \frac{3}{10} \text{ mm} \)

9. ERROR ANALYSIS Describe and correct the error in finding the missing length of the triangle.

   \[ a^2 + b^2 = c^2 \]
   \[ 6^2 + 20^2 = c^2 \]
   \[ 436 = c^2 \]
   \[ \sqrt{436} = c \]

10. TRIPOD The center of the tripod forms a 90° angle with the ground. Find the length of the support leg to the nearest tenth of an inch.
11. **TELEVISIONS** Televisions are advertised by the lengths of their diagonals. Approximate the length of the diagonal of the television to the nearest inch.

12. **TENNIS** A tennis player asks the referee a question. The sound of the player’s voice only travels 50 feet. Can the referee hear the question? Explain.

13. **GOLF** The figure shows the location of a golf ball after a tee shot. How many feet from the hole is the ball?

14. **SNOWBALLS** You and a friend throw snowballs at each other. You are 20 feet north and 15 feet east of your house. Your friend is 25 feet east and 10 feet north of your house. How far apart are you and your friend?

15. **PRECISION** The legs of a right triangle have lengths of 28 meters and 21 meters. The hypotenuse has a length of $5x$ meters.
   a. Write an equation to solve for $x$.
   b. Describe how to solve the equation by factoring and by taking a square root. Which method do you prefer? Explain.
   c. What is the value of $x$?

16. **Structure** The side lengths of a right triangle are three consecutive integers.
   a. Write an expression that represents each side length. Which side length represents the hypotenuse?
   b. Write and solve an equation to find the three integers.

---

**Fair Game Review** What you learned in previous grades & lessons

Graph the function. Compare the graph to the graph of $y = x^2$.  
(Section 8.3)

17. $y = -2x^2 + 4$  
18. $y = -x^2 - 6$  
19. $y = 3x^2 + 8$

20. **MULTIPLE CHOICE** Which polynomial is equivalent to $(x^2 - 3x + 1) - (-2x^2 + x - 4)$?
   (Section 7.2)
   
   A) $3x^2 - 4x - 3$  
   B) $-x^2 - 2x - 5$  
   C) $3x^2 - 4x + 5$  
   D) $-x^2 + 4x + 3$

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Section 10.3  The Pythagorean Theorem  525
Using the Pythagorean Theorem

Essential Question In what other ways can you use the Pythagorean Theorem?

The converse of a statement switches the hypothesis and the conclusion.

**Statement:**
If \( p \), then \( q \).

**Converse of the statement:**
If \( q \), then \( p \).

1. **ACTIVITY: Analyzing Converse of Statements**

Work with a partner. Write the converse of the true statement. Determine whether the converse is true or false. If it is false, give a counterexample.

a. Sample: If \( a = b \), then \( a^2 = b^2 \).
   Converse: If \( a^2 = b^2 \), then \( a = b \).
   The converse is false. A counterexample is \( a = -2 \) and \( b = 2 \).

b. If two nonvertical lines have the same slope, then the lines are parallel.

c. If a sequence has a common difference, then it is an arithmetic sequence.

d. If \( a \) and \( b \) are rational numbers, then \( a + b \) is a rational number.

Is the converse of a true statement always true? always false? Explain.

2. **ACTIVITY: The Converse of the Pythagorean Theorem**

Work with a partner. The converse of the Pythagorean Theorem states: “If the equation \( a^2 + b^2 = c^2 \) is true for the side lengths of a triangle, then the triangle is a right triangle.”

a. Do you think the converse of the Pythagorean Theorem is true or false? How could you use deductive reasoning to support your answer?

b. Consider \( \triangle DEF \) with side lengths \( a, b, \) and \( c \), such that \( a^2 + b^2 = c^2 \). Also consider \( \triangle JKL \) with leg lengths \( a \) and \( b \), where \( \angle K = 90^\circ \).
   - What does the Pythagorean Theorem tell you about \( \triangle JKL \)?
   - What does this tell you about \( c \) and \( x \)?
   - What does this tell you about \( \triangle DEF \) and \( \triangle JKL \)?
   - What does this tell you about \( \angle E \)?
   - What can you conclude?
Work with a partner. Follow the steps below to write a formula that you can use to find the distance between any two points in a coordinate plane.

Step 1: Choose two points in the coordinate plane that do not lie on the same horizontal or vertical line. Label the points \((x_1, y_1)\) and \((x_2, y_2)\).

Step 2: Draw a line segment connecting the points. This will be the hypotenuse of a right triangle.

Step 3: Draw horizontal and vertical line segments from the points to form the legs of the right triangle.

Step 4: Use the \(x\)-coordinates to write an expression for the length of the horizontal leg.

Step 5: Use the \(y\)-coordinates to write an expression for the length of the vertical leg.

Step 6: Substitute the expressions for the lengths of the legs into the Pythagorean Theorem.

Step 7: Solve the equation in Step 6 for the hypotenuse \(c\).

What does the length of the hypotenuse tell you about the two points?

What Is Your Answer?

4. **IN YOUR OWN WORDS** In what other ways can you use the Pythagorean Theorem?

5. What kind of real-life problems do you think the converse of the Pythagorean Theorem can help you solve?

Use what you learned about the converse of a true statement to complete Exercises 3 and 4 on page 530.
**Key Ideas**

**Converse of the Pythagorean Theorem**
If the equation $a^2 + b^2 = c^2$ is true for the side lengths of a triangle, then the triangle is a right triangle.

**Study Tip**
A Pythagorean triple is a set of three positive integers $a$, $b$, and $c$, where $a^2 + b^2 = c^2$.

**Common Error**
When using the converse of the Pythagorean Theorem, always substitute the length of the longest side for $c$.

**EXAMPLE 1**

Identifying a Right Triangle

Tell whether each triangle is a right triangle.

**a.**
- $a^2 + b^2 = c^2$
- $7^2 + 24^2 = 25^2$
- $49 + 576 = 625$
- $625 = 625$ ✓

It is a right triangle.

**b.**
- $a^2 + b^2 = c^2$
- $40^2 + 80^2 = 90^2$
- $1600 + 6400 = 8100$
- $8000 ≠ 8100$ ✗

It is not a right triangle.

**On Your Own**

Tell whether the triangle with the given side lengths is a right triangle.

1. 15 cm, 10 cm, 18 cm
2. 50 yd, 40 yd, 30 yd

**Key Idea**

**Distance Formula**
The distance $d$ between any two points $(x_1, y_1)$ and $(x_2, y_2)$ is given by the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. 

---

**Check It Out**
Lesson Tutorials at BigIdeasMath.com
EXAMPLE 2 Finding the Distance Between Two Points

Find the distance between \((-3, 5)\) and \((2, -1)\).

Let \((x_1, y_1) = (-3, 5)\) and \((x_2, y_2) = (2, -1)\).

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
d = \sqrt{[2 - (-3)]^2 + (-1 - 5)^2}
\]

\[
d = \sqrt{5^2 + (-6)^2}
\]

\[
d = \sqrt{25 + 36}
\]

\[
d = \sqrt{61}
\]

EXAMPLE 3 Real-Life Application

A football coach designs a passing play in which a receiver runs down the field, makes a 90° turn, and runs to the corner of the end zone. A receiver runs the play as shown. Did the receiver run the play as designed? Each unit of the grid represents 10 feet.

Use the distance formula to find the lengths of the three sides.

\[
d_1 = \sqrt{(60 - 20)^2 + (50 - 10)^2} = \sqrt{40^2 + 40^2} = \sqrt{3200} \text{ feet}
\]

\[
d_2 = \sqrt{(80 - 20)^2 + (-30 - 10)^2} = \sqrt{60^2 + (-40)^2} = \sqrt{5200} \text{ feet}
\]

\[
d_3 = \sqrt{(80 - 60)^2 + (-30 - 50)^2} = \sqrt{20^2 + (-80)^2} = \sqrt{6800} \text{ feet}
\]

Use the converse of the Pythagorean Theorem to determine if the side lengths form a right triangle.

\[
(\sqrt{3200})^2 + (\sqrt{5200})^2 \stackrel{?}{=} (\sqrt{6800})^2
\]

\[
3200 + 5200 \stackrel{?}{=} 6800
\]

\[
8400 \neq 6800
\]

It is not a right triangle. So, the receiver did not make a 90° turn.

The receiver did not run the play as designed.

On Your Own

Find the distance between the two points.

3. \((0, 4), (5, 2)\)  4. \((-1, 3), (4, -8)\)  5. \((-10, -6), (6, 2)\)

6. WHAT IF? In Example 3, the receiver made the turn at \((30, 20)\). Did the receiver run the play as designed? Explain.
1. **Writing** Describe two ways to find the distance between two points in a coordinate plane.

2. **Which One Doesn’t Belong?** Which set of numbers does not belong with the other three? Explain your reasoning.

   - 3, 4, 5
   - 8, 15, 17
   - 18, 22, 29
   - 9, 40, 41

---

**Practice and Problem Solving**

Write the converse of the true statement. Determine whether the converse is true or false.

3. If $a$ is an even number, then $a^2$ is even.
4. If $a$ is positive, then $|a| = a$.

Tell whether the triangle with the given side lengths is a right triangle.

5. $9$ m, $12$ m, $15$ m
6. $7$ ft, $11$ ft, $8$ ft
7. $3.7$ cm, $3.5$ cm, $1.2$ cm

8. $16$ in., $18$ in., $24$ in.
9. $30$ yd, $22$ yd, $15$ yd
10. $8$ mm, $15$ mm, $17$ mm

Find the distance between the two points.

11. $(4, -3), (-2, -5)$
12. $(1, 1), (4, 5)$
13. $(-7, -1), (-4, 8)$
14. $(1, 3), (7, 7)$
15. $(2, -8), (4, -1)$
16. $(-7, 4), (-3, 2)$

17. **Error Analysis** Describe and correct the error in finding the distance between the points $(-4, -3)$ and $(2, -1)$.

   $$d = \sqrt{(2 - (-4))^2 - [-1 - (-3)]^2}$$
   $$= \sqrt{36 - 4}$$
   $$= 4\sqrt{2}$$

18. **Construction** A post and beam frame for a shed is shown in the diagram. Does the brace form a right triangle with the post and beam? Explain.
Tell whether the set of measurements can be the side lengths of a right triangle.

19. $5\sqrt{5}$, 10, 15  
20. $7, 3\sqrt{10}, 6$  
21. 21, 72, 75

22. **REASONING** Plot the points $(-1, -2), (2, 1)$, and $(-3, 6)$ in a coordinate plane. Are the points the vertices of a right triangle? Explain.

23. **GEOCACHING** You spend the day looking for hidden containers in a wooded area using a global positioning system (GPS). You park your car on the side of the road and then locate Container 1 and Container 2 before going back to the car. Does your path form a right triangle? Explain. Each unit of the grid represents 10 yards.

24. **REASONING** Your teacher wants the class to find the distance between the two points (3, 2) and (8, 6). You choose (3, 2) for $(x_1, y_1)$ and your friend chooses (8, 6) for $(x_2, y_2)$. Do you and your friend obtain the same answer? Justify your answer.

25. **AIRPORT** Which plane is closer to the base of the airport tower? Explain.

26. Structure **Consider** the two points $(x_1, y_1)$ and $(x_2, y_2)$ in the coordinate plane. How can you find the point $(x_m, y_m)$ located in the middle of the two given points? Justify your answer using the distance formula.

![Diagram](https://via.placeholder.com/150)

**Fair Game Review** What you learned in previous grades & lessons.

Solve the equation using the quadratic formula. *(Section 9.4)*

27. $2x^2 - 5x + 3 = 0$  
28. $2x^2 - x - 1 = 0$  
29. $x^2 + 3x + 5 = 0$

30. **MULTIPLE CHOICE** Which point is the focus of the graph of $y = 2x^2$? *(Section 8.2)*

A) $\left(0, \frac{1}{8}\right)$  
B) $\left(0, \frac{1}{4}\right)$  
C) $\left(0, \frac{1}{2}\right)$  
D) $\left(0, -\frac{1}{8}\right)$
10.3–10.4 Quiz

Find the missing length of the triangle.  (Section 10.3)

1. \[\text{9 ft} \quad \text{40 ft} \quad c\]
2. \[a \quad \text{53 in.} \quad \text{45 in.}\]
3. \[1.6 \text{ cm} \quad \text{6.5 cm} \quad b\]
4. \[65 \text{ yd} \quad 72 \text{ yd} \quad c\]

Tell whether the triangle with the given side lengths is a right triangle.  (Section 10.4)

5. \[\text{53 ft} \quad \text{46 ft} \quad \text{28 ft}\]
6. \[\text{16 m} \quad \text{12 m} \quad \text{20 m}\]

Find the distance between the two points.  (Section 10.4)

7. \((-3, -1), (-1, -5)\)
8. \((-4, 2), (5, 1)\)
9. \((1, -2), (4, -5)\)
10. \((-1, 1), (7, 4)\)
11. \((-6, 5), (-4, -6)\)
12. \((-1, 4), (1, 3)\)

13. FABRIC  You cut a rectangular piece of fabric in half along the diagonal. The fabric measures 28 inches wide and \(1 \frac{1}{4}\) yards long. What is the length (in inches) of the diagonal?  (Section 10.3)

Use the figure to answer Exercises 14–17.  (Section 10.4)

14. How far is the cabin from the peak?
15. How far is the fire tower from the lake?
16. How far is the lake from the peak?
17. You are standing at \((-5, -6)\). How far are you from the lake?
10 Chapter Review

Review Key Vocabulary
- square root function, p. 504
- simplest form of a radical expression, p. 508
- rationalizing the denominator, p. 508
- conjugates, p. 509
- square root equation, p. 512
- extraneous solution, p. 513
- theorem, p. 520
- legs, p. 522
- hypotenuse, p. 522
- Pythagorean Theorem, p. 522
- distance formula, p. 528

Review Examples and Exercises

10.1 Graphing Square Root Functions (pp. 502–509)

a. Graph \( y = \sqrt{x} - 1 \). Describe the domain and range. Compare the graph to the graph of \( y = \sqrt{x} \).

Step 1: Make a table of values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>4</th>
<th>9</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Step 2: Plot the ordered pairs.
Step 3: Draw a smooth curve through the points.

- The domain is \( x \geq 0 \). The range is \( y \geq -1 \). The graph of \( y = \sqrt{x} - 1 \) is a translation 1 unit down of the graph of \( y = \sqrt{x} \).

b. Graph \( y = \sqrt{x + 2} \). Describe the domain and range. Compare the graph to the graph of \( y = \sqrt{x} \).

Step 1: Make a table of values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0</td>
<td>1</td>
<td>1.4</td>
<td>1.7</td>
<td>2</td>
</tr>
</tbody>
</table>

Step 2: Plot the ordered pairs.
Step 3: Draw a smooth curve through the points.

- The domain is \( x \geq -2 \). The range is \( y \geq 0 \). The graph of \( y = \sqrt{x + 2} \) is a translation 2 units to the left of the graph of \( y = \sqrt{x} \).

Exercises

Graph the function. Describe the domain and range. Compare the graph to the graph of \( y = \sqrt{x} \).

1. \( y = \sqrt{x} + 7 \)
2. \( y = \sqrt{x - 6} \)
3. \( y = -\sqrt{x + 3} - 1 \)
10.2 Solving Square Root Equations  \( (pp. \ 510\ – \ 517) \)

Solve \( \sqrt{12 - x} = x \).

\[
\sqrt{12 - x} = x \\
(x - \sqrt{12 - x})^2 = x^2 \\
12 - x = x^2 \\
0 = x^2 + x - 12 \\
0 = (x - 3)(x + 4) \\
(x - 3) = 0 \quad \text{or} \quad (x + 4) = 0 \\
x = 3 \quad \text{or} \quad x = -4
\]

Check \( \sqrt{12 - 3} = 3 \) \quad \text{Substitute for } x. \\
\( \sqrt{12 - (-4)} = -4 \)

\[
\sqrt{9} = 3 \quad \text{Simplify.} \\
\sqrt{16} = 4 \\
3 = 3 \quad \checkmark \\
4 \neq -4 \quad \times
\]

Because \( x = -4 \) does not check in the original equation, it is an extraneous solution. The only solution is \( x = 3 \).

Exercises

Solve the equation. Check your solution.

4. \( 8 + \sqrt{x} = 18 \)  \hspace{1cm} 5. \( \sqrt{x - 1} + 9 = 15 \)
6. \( \sqrt{5x - 9} = \sqrt{4x} \)  \hspace{1cm} 7. \( x = \sqrt{3x + 4} \)

10.3 The Pythagorean Theorem  \( (pp. \ 520\ – \ 525) \)

Find the length of the hypotenuse of the triangle.

\[
a^2 + b^2 = c^2 \quad \text{Write the Pythagorean Theorem.} \\
10^2 + 24^2 = c^2 \quad \text{Substitute.} \\
100 + 576 = c^2 \quad \text{Evaluate powers.} \\
676 = c^2 \quad \text{Add.} \\
\sqrt{676} = \sqrt{c^2} \quad \text{Take positive square root of each side.} \\
26 = c \quad \text{Simplify.}
\]

The length of the hypotenuse is 26 meters.
Using the Pythagorean Theorem (pp. 526–531)

a. Is the triangle formed by the rope and the tent a right triangle?

\[ a^2 + b^2 = c^2 \]

\[ 64^2 + 48^2 = 80^2 \]

\[ 4096 + 2304 = 6400 \]

\[ 6400 = 6400 \]

\[ \checkmark \]

It is a right triangle.

b. Find the distance between \((-3, 1)\) and \((4, 7)\).

Let \((x_1, y_1) = (-3, 1)\) and \((x_2, y_2) = (4, 7)\).

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\[ = \sqrt{(4 - (-3))^2 + (7 - 1)^2} \]

\[ = \sqrt{7^2 + 6^2} \]

\[ = \sqrt{49 + 36} \]

\[ = \sqrt{85} \]

Exercises

Tell whether the triangle is a right triangle.

10. \[ \triangle \]

Find the distance between the two points.

12. \((-2, -5), (3, 5)\)

13. \((-4, 7), (4, 0)\)
Graph the function. Describe the domain and range. Compare the graph to the graph of \( y = \sqrt{x} \).

1. \( y = \sqrt{x} - 6 \)
2. \( y = \sqrt{x} + 10 \)
3. \( y = -\sqrt{x} - 2 + 3 \)

Solve the equation.
4. \( 9 - \sqrt{x} = 3 \)
5. \( \sqrt{2x - 7} - 3 = 6 \)
6. \( \sqrt{8x - 21} = \sqrt{18 - 5x} \)
7. \( x + 5 = \sqrt{7x} + 53 \)

Find the missing length of the triangle.
8. \( \text{c} \)
\( 18 \text{ m} \)
\( 24 \text{ m} \)

9. \( b \)
\( 21 \text{ mm} \)
\( 35 \text{ mm} \)

Tell whether the triangle is a right triangle.
10. \( 39 \text{ m} \)
\( 35 \text{ m} \)
\( 15 \text{ m} \)
11. \( 42 \text{ ft} \)
\( 40 \text{ ft} \)
\( 58 \text{ ft} \)

Find the distance between the two points.
12. \( (-2, 3), (6, 9) \)
13. \( (0, -5), (4, 1) \)
14. \( (-3, -4), (2, -7) \)

15. **ROLLER COASTER** The velocity \( v \) (in meters per second) of a roller coaster at the bottom of a hill is given by \( v = \sqrt{19.6h} \), where \( h \) is the height of the hill (in meters). (a) Graph the function. Describe the domain and range. (b) How tall must the hill be for the velocity of the roller coaster at the bottom of the hill to be at least 28 meters per second?

16. **FINANCE** The average annual interest rate \( r \) (in decimal form) that an investment earns over 2 years is given by \( r = \sqrt{\frac{V_2}{V_0}} - 1 \), where \( V_0 \) is the initial investment and \( V_2 \) is the value of the investment after 2 years. You initially invest $800 which earns an average annual interest of 6% over 2 years. What is the value of \( V_2 \)?

17. **BUTTERFLY** Approximate the wingspan of the butterfly.
1. Which function is shown in the graph? 
   (F.IF.7b)

![Graph Image]

A. \( y = -\sqrt{x + 1} + 2 \)
B. \( y = -\sqrt{x - 1} + 2 \)
C. \( y = -\sqrt{x + 2} + 1 \)
D. \( y = -\sqrt{x - 2} + 1 \)

2. Which number is equivalent to \( \frac{8^{10} \cdot 2^{-5}}{16^4} \)? (N.RN.2)
   
   F. 1
   H. \( 2^{24} \)
   G. \( 2^9 \)
   I. \( 2^{-11} \)

3. What value of \( x \) makes the equation below true? (N.RN.2)
   
   \[ x = \sqrt{2x + 8} \]

4. A line with a slope of \(-2\) passes through the point \((1, -6)\). Which of the following is not a point on the line? (A.CED.2)
   
   A. \((-8, 12)\)
   B. \((-4, 4)\)
   C. \((4, -4)\)
   D. \((8, -20)\)

5. What value(s) of \( x \) make the equation below true? (N.RN.2)
   
   \[ \sqrt{2x - 4} = x - 2 \]

F. Only \(-2\)
G. 2 and 4
H. Only \(4\)
I. \(-2\) and \(-4\)
6. What is the focus of the parabola? \( (F.IF.4) \)

![Graph of a parabola]

A. \( \left( 0, \frac{1}{16} \right) \)  
B. \( (0, 16) \)  
C. \( \left( 0, \frac{-1}{16} \right) \)  
D. \( (0, -16) \)  

7. The range of the function \( y = 6x - 8 \) is all real numbers from 1 to 10. What is the domain of the function? \( (F.IF.1) \)

F. all real numbers from 1.5 to 3  
G. all real numbers from \(-2\) to 52  
H. all integers from \(-2\) to 52  
I. all real numbers

8. What is the distance between the two points in the coordinate plane? \( (8.G.8) \)

![Graph showing two points]

9. Which ordered pair is a solution of the system of inequalities shown in the graph? \( (A.REI.12) \)

A. \( (1, 0) \)  
B. \( (0, 1) \)  
C. \( (0, -1) \)  
D. \( (2, 0) \)
10. Two nature trails are shown below. Which trail is longer? By how much? Explain your reasoning. \((8.G.7)\)

11. The solution of which inequality is shown in the graph below? \((A.REI.3)\)

F. \(5x - 7 \geq 3\)  
G. \(4x + 3 \leq 11\)  
H. \(12 - 3x < 6\)  
I. \(10 - 2x > 6\)

12. Tom was graphing \(y = \sqrt{x+2} - 1\). His work is shown below. \((F.IF.7b)\)

What should Tom do to correct the error that he made?

A. Shift the graph 1 unit down and 1 unit left.

B. Shift the graph 4 units left.

C. Shift the graph 3 units up and 3 units left.

D. Shift the graph 1 unit down and 3 units left.

13. What is the vertex of the graph of \(y = 2x^2 - 4x + 6\)? \((F.IF.4)\)

F. \((-1, 12)\)  
G. \((-2, 22)\)  
H. \((1, 4)\)  
I. \((2, 6)\)