Before

In previous chapters you’ve . . .
• Written and evaluated variable expressions
• Used the distributive property to simplify variable expressions
• Solved one-step equations

Now

In Chapter 3 you’ll study . . .
• Writing and solving two-step equations
• Using the distributive property to solve equations
• Writing and solving inequalities
• Graphing inequalities on a number line

Why?

So you can solve real-world problems about . . .
• rafting, p. 123
• shopping, p. 133
• astronauts, p. 142
• skiing, p. 144
• in-line skates, p. 149
• amusement parks, p. 153
• advertising, p. 154
Wind Power These wind turbines near Palm Springs, California, produce power when the wind turns their blades. In this chapter, you will use inequalities to describe quantities like wind speed.

What do you think? Suppose a turbine begins producing power when the wind speed is at least 10 miles per hour, and shuts down when the wind speed exceeds 65 miles per hour. Plot 10 and 65 on a number line. Shade the number line to show the speeds at which the turbine produces power.
Chapter Prerequisite Skills

PREREQUISITE SKILLS QUIZ

Preparing for Success To prepare for success in this chapter, test your knowledge of these concepts and skills. You may want to look at the pages referred to in blue for additional review.

1. **Vocabulary** Explain what an equation is. Then give an example of an equation with a variable in it.

Use the distributive property to write an equivalent variable expression.

2. $9(x - 4)$
3. $8(z - 7)$
4. $-6(-m + 12)$
5. $-10(n - 5)$

Simplify the expression.

6. $c + 4 - c$
7. $9b - 12b + 3$
8. $4(a + 2) + a$
9. $2(2d + 5 + d)$

Solve the equation. Check your solution.

10. $x + 13 = 7$
11. $\frac{h}{6} = -8$
12. $q - 9.6 = 2$
13. $65 = -13b$

NOTETAKING STRATEGIES

**SUMMARIZING** At the end of each lesson, summarize the main idea of the lesson in your notes. Include important details.

Lesson 2.6 Solving Equations Using Multiplication or Division

Main Idea: Multiplying or dividing each side of an equation by the same nonzero number results in an equivalent equation.

Use division to solve multiplication equations. Use multiplication to solve division equations.

\[
\begin{align*}
2x &= 6 \\
\frac{2x}{2} &= \frac{6}{2} \\
x &= 3
\end{align*}
\]

\[
\begin{align*}
x &= 10 \\
5\left(\frac{x}{5}\right) &= 5(10) \\
x &= 50
\end{align*}
\]

You may find this strategy helpful in Lesson 3.6 when you solve multi-step inequalities.
Use algebra tiles to solve \(3x + 6 = 12\).

1. Model \(3x + 6 = 12\) using algebra tiles.

2. Work backward to undo the operations performed on \(x\). Begin by removing six 1-tiles from each side.

3. Divide the remaining tiles into three equal groups. Each \(x\)-tile is equal to two 1-tiles. So, the solution is 2.

**Draw Conclusions**

Use algebra tiles to model and solve the equation.

1. \(1 + 2x = 9\)
2. \(4x + 1 = 5\)
3. \(2x + 2 = 8\)
4. \(9 = 2x + 5\)
5. \(11 = 2 + 3x\)
6. \(5x + 3 = 8\)

7. **Critical Thinking** What property of equality is used in Step 2? in Step 3?

8. **Writing** For each algebra-tile model shown above, write a corresponding algebraic equation.

9. **Interpret** Describe the steps you would take to solve \(2x + 1 = 5\) without using algebra tiles.

10. **Critical Thinking** The equation above, \(3x + 6 = 12\), involves two operations performed on \(x\): multiplication by 3 followed by addition of 6. What operations were used to isolate \(x\)? In what order were the operations used? How does solving a two-step equation require you to work backward?
Solving Two-Step Equations

You solved one-step equations.

You’ll solve two-step equations.

So you can find the cost of a rafting trip, as in Ex. 21.

Drum Set You are buying a drum set that costs $495. The music store lets you make a down payment. You can pay the remaining cost in three equal monthly payments with no interest charged. You make a down payment of $150. How much is each monthly payment? In Example 4, you will see how to answer this question by writing and solving a two-step equation.

You can solve a two-step equation by using two inverse operations.

Example 1 Using Subtraction and Division to Solve

Solve \(3x + 7 = -5\). Check your solution.

\[
3x + 7 = -5 \\
3x + 7 - 7 = -5 - 7 \\
3x = -12 \\
\frac{3x}{3} = \frac{-12}{3} \\
x = -4
\]

Answer The solution is \(-4\).

✓ Check \(3x + 7 = -5\)

\[
3(-4) + 7 = -5 \\
-12 + 7 = -5 \\
-5 = -5 \checkmark
\]

Checkpoint

Solve the equation. Check your solution.

1. \(4x + 1 = 5\)  
2. \(3n + 8 = 2\)  
3. \(1 = 2r + 9\)  
4. \(2 = 6h + 20\)  
5. Critical Thinking How is solving \(3x - 7 = -5\) different from solving \(3x + 7 = -5\)?
Example 2  Using Addition and Multiplication to Solve

Solve \( \frac{x}{2} - 3 = 1 \). Check your solution.

\[ \frac{x}{2} - 3 = 1 \]  
Write original equation.

\[ \frac{x}{2} - 3 + 3 = 1 + 3 \]  
Add 3 to each side.

\[ \frac{x}{2} = 4 \]  
Simplify.

\[ 2 \left( \frac{x}{2} \right) = 2(4) \]  
Multiply each side by 2.

\[ x = 8 \]  
Simplify.

Answer  The solution is 8.

\( \checkmark \) Check \( \frac{x}{2} - 3 = 1 \)  
Write original equation.

\[ \frac{8}{2} - 3 \neq 1 \]  
Substitute 8 for \( x \).

\[ 1 = 1 \checkmark \]  
Solution checks.

Check

Solve the equation. Check your solution.

6. \( \frac{b}{4} - 8 = 1 \)  
7. \( \frac{c}{6} - 2 = 6 \)  
8. \( 2 = \frac{d}{5} - 1 \)  
9. \( 12 = \frac{f}{2} - 8 \)

Example 3  Solving an Equation with Negative Coefficients

Solve \( 7 - 4y = 19 \). Check your solution.

\[ 7 - 4y = 19 \]  
Write original equation.

\[ 7 - 4y - 7 = 19 - 7 \]  
Subtract 7 from each side.

\[ -4y = 12 \]  
Simplify.

\[ \frac{-4y}{-4} = \frac{12}{-4} \]  
Divide each side by \(-4\).

\[ y = -3 \]  
Simplify.

Answer  The solution is \(-3\).

\( \checkmark \) Check \( 7 - 4y = 19 \)  
Write original equation.

\[ 7 - 4(-3) \neq 19 \]  
Substitute \(-3\) for \( y \).

\[ 19 = 19 \checkmark \]  
Solution checks.

Check

Solve the equation. Check your solution.

10. \( 12 - 4s = -12 \)  
11. \( 6 - 2m = 8 \)  
12. \( -2 = 5 - n \)
Guided Practice

1. Copy and complete: You can use two _ ? _ operations to solve a two-step equation.
2. Describe the steps you would use to solve the equation $9 + 2s = 15$.

Skill Check

Solve the equation. Check your solution.

3. $5c + 6 = 31$
4. $-2 = \frac{t}{3} - 11$
5. $-9z + 4 = -5$
6. $-8 - 8d = 64$

Guided Problem Solving

7. Car Repair The total cost of repairing a car is the sum of the amount paid for parts and the amount paid for labor. You paid $78 for parts and $45 for each hour of labor. The total cost to repair the car was $168. How many hours did it take to repair the car?

1) Copy and complete the verbal model.

\[
\text{Total cost for repairs} = \ ? + \text{Cost for each hour of labor} \cdot \ ?
\]

2) Let $h$ represent the number of hours spent on labor. Write an equation based on your verbal model.

3) Solve the equation to find how many hours it took to repair the car.
Solve the equation. Check your solution.

8. $12k + 7 = 31$
9. $13n + 42 = 81$
10. $56 = 17p - 29$
11. $\frac{w}{4} - 21 = -3$
12. $\frac{h}{9} - 19 = -10$
13. $\frac{d}{12} + 25 = 29$
14. $12 = \frac{a}{36} + 17$
15. $18 - r = 42$
16. $80 = 23 - 3v$
17. $-2q - 63 = 47$
18. $-\frac{x}{2} + 4 = 12$
19. $-5 = -19 - \frac{x}{7}$

20. **Driving** Your family is taking a long-distance car trip. You begin with 16 gallons of gasoline in the fuel tank. Your car uses 3 gallons of gasoline per hour of driving. You will stop to refuel when there is exactly 1 gallon of gasoline remaining in the tank.

   a. **Analyze** List the given information and what you need to find.
   b. Write a verbal model. Then write an equation based on your model.
   c. After how many hours will you need to stop to refuel? Justify your solution by making a table.

21. **Rafting** A group of 9 friends takes a white-water rafting trip. The total price of the trip before any discounts is $810. Each person in the group receives a student discount. The total price with the discount is $729. How much is the discount per person?

22. **Dog Walking** Jasmine walks three dogs after school. She brings dog treats for the dogs. She gives 4 treats to Tucker. Then she splits the remaining treats between Bogey and Murphy, who get 3 treats each. How many treats did Jasmine start with? Solve the problem by working backward, and by writing and solving an equation.

Write the verbal sentence as an equation. Then solve the equation.

23. Five minus the product of 2 and a number is 7.
24. Thirty-two minus the product of 9 and a number is 140.
25. Thirteen plus the product of 6 and a number is 67.
26. Negative 8 minus the product of 3 and a number is 19.

27. **Extended Problem Solving** Your class has raised $755 for a hunger relief organization. The organization provides farm animals that people can use to produce food. Your class plans to buy animals for a family recovering from an earthquake.

   a. **Calculate** One heifer (a young cow) costs $500, and each flock of chicks costs $20. If your class buys one heifer, how many flocks of chicks can your class buy?
   b. **Calculate** Your class can also buy pigs for $120 each. If your class buys a heifer for $500, how many pigs can your class buy?
   c. **Interpret and Apply** If your class decides to buy the heifer and pigs as described in part (b), does your class have enough money to also buy a flock of chicks? Explain your reasoning.
Solve the equation. Check your solution.

28. $54.7 = -9.3n + 8.2$
29. $-5.7 + 2.6d = -14.02$
30. $3.2r + 14.7 = -6.74$
31. $9.1 = \frac{k}{3.7} + 4.1$
32. $11.3 - \frac{p}{2.8} = 1.5$
33. $-6.8 - \frac{c}{1.2} = -2.9$

34. **Compare and Contrast** Your friend solved the equation $18 - 2x = -36$ by first adding $2x$ to each side of the equation. You solved the equation by subtracting 18 from each side as the first step. Compare and contrast the two methods. What do you notice?

35. **Class Trip** You are saving money for a class trip to Washington, D.C. You need $850 for the trip. You have saved $278. You can save an additional $50 each month.

   a. Write a variable expression to represent the total amount of money you have saved after $m$ months. Evaluate your expression for whole-number values of $m$. Record your results in a table like the one shown.

<table>
<thead>
<tr>
<th>Number of months from now</th>
<th>Amount of money saved</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$278</td>
</tr>
<tr>
<td>1</td>
<td>?</td>
</tr>
<tr>
<td>2</td>
<td>?</td>
</tr>
<tr>
<td>3</td>
<td>?</td>
</tr>
<tr>
<td>4</td>
<td>?</td>
</tr>
</tbody>
</table>

   b. **Analyze** Use the data in your table to make a scatter plot. Put months on the horizontal axis and savings on the vertical axis. What pattern do you notice in your graph? How can you use the graph to find the number of months it will take you to save enough money for the trip?

c. Write and solve an equation to find the number of months it will take you to save enough money for the trip.

d. **Compare** List some advantages and disadvantages of the methods you used in parts (a), (b), and (c).

36. **Challenge** Solve $\frac{x + 2}{4} = 2$. Explain how you solved the equation and how you know your solution is correct.

**Mixed Review**

Use the distributive property to write an equivalent variable expression. *Lesson 2.2*

37. $11(6z + 14)$
38. $-9(2x + 12)$
39. $12(3 - 5y)$
40. $8(4 - 7w)$

**Algebra Basics** Solve the equation. Check your solution. *Lesson 2.5*

41. $c + 12 = 23$
42. $b + 14 = 91$
43. $x - 17 = -45$
44. $d - 22 = -43$

**Standardized Test Practice**

45. **Multiple Choice** What is the solution of the equation $15y - 63 = 57$?
   A. $-8$  
   B. $8$  
   C. $9$  
   D. $10$

46. **Short Response** You purchase a video game system for $150. You make a down payment of $25. You pay the rest of the money you owe in 5 equal monthly payments with no interest. How much is each monthly payment? Show how you found your answer.
Review Vocabulary

like terms, p. 78

School Spirit  Your school’s basketball team is playing in the championship game. For the game, the cheerleaders want to buy a banner that costs $47. They also want to buy small items to give to students in the stands. Pompoms cost $5.20 each. Noisemakers cost $.80 each. The cheerleaders have a total budget of $375 for the game. If they buy equal numbers of pompoms and noisemakers, how many can they afford to buy?

Example 1  Writing and Solving an Equation

Find how many pompoms and noisemakers the cheerleaders can afford to buy, as described above.

Solution

Let \( n \) represent the number of pompoms and the number of noisemakers. Then \( 5.20n \) represents the cost of \( n \) pompoms, and \( 0.80n \) represents the cost of \( n \) noisemakers. Write a verbal model.

\[
\begin{align*}
\text{Cost of } n \text{ pompoms} & \quad + \quad \text{Cost of } n \text{ noisemakers} & \quad + \quad \text{Cost of banner} & \quad = \quad \text{Total budget} \\
5.20n & \quad + \quad 0.80n & \quad + \quad 47 & \quad = \quad 375 \\
6.00n & \quad + \quad 47 & \quad = \quad 375 \\
6n & \quad + \quad 47 & \quad - \quad 47 & \quad = \quad 375 \quad - \quad 47 \\
6n & \quad = \quad 328 \\
\frac{6n}{6} & \quad = \quad \frac{328}{6} \\
n & \quad = \quad 54 \frac{2}{3}
\end{align*}
\]


Answer  The answer must be a whole number. Round down so the budget is not exceeded. The cheerleaders can afford to buy 54 pompoms and 54 noisemakers.
**Distributive Property** You can use the distributive property to solve equations involving parentheses.

### Example 2  Solving Equations Using the Distributive Property

**Solve the equation.**

**a.** \(-21 = 7(3 - x)\)

**Solution**

\[-21 = 7(3 - x)\]  Write original equation.

\[-21 = 21 - 7x\]  Distributive property

\[-21 - 21 = 21 - 7x - 21\]  Subtract 21 from each side.

\[-42 = -7x\]  Simplify.

\[-\frac{42}{-7} = -\frac{7x}{-7}\]  Divide each side by \(-7\).

\[6 = x\]  Simplify.

**Answer** The solution is 6.

**b.** \(-3(8 - 4x) = 12\)

**Solution**

\[-3(8 - 4x) = 12\]  Write original equation.

\[-24 + 12x = 12\]  Distributive property

\[-24 + 12x + 24 = 12 + 24\]  Add 24 to each side.

\[12x = 36\]  Simplify.

\[\frac{12x}{12} = \frac{36}{12}\]  Divide each side by 12.

\[x = 3\]  Simplify.

**Answer** The solution is 3.

### Example 3  Combining Like Terms After Distributing

**Solve** \(5x - 2(x - 1) = 8\).

\[5x - 2(x - 1) = 8\]  Write original equation.

\[5x - 2x + 2 = 8\]  Distributive property

\[3x + 2 = 8\]  Combine like terms.

\[3x + 2 - 2 = 8 - 2\]  Subtract 2 from each side.

\[3x = 6\]  Simplify.

\[\frac{3x}{3} = \frac{6}{3}\]  Divide each side by 3.

\[x = 2\]  Simplify.

**Checkpoint**

**Solve the equation. Check your solution.**

1. \(3n - 40 + 2n = 15\)
2. \(2(s - 1) = 6\)
3. \(13 = 2y - 3(y + 4)\)
Guided Practice

1. What property do you use when you rewrite the equation $6(x + 1) = 12$ as $6x + 6 = 12$?

2. Identify the like terms you would combine to solve the equation $–3x + 5 – 2x + 8 = 12$.

Skill Check

Solve the equation. Check your solution.

3. $4 + x + 7 = 10$
4. $3x + 2x = 25$
5. $21 = 4x - 9 - x$
6. $3(x + 1) = 6$
7. $16 = 8(x - 1)$
8. $5 + 2(x - 2) = 19$

Guided Problem Solving

9. Geometry The perimeter of the rectangle shown is 28 units. The length is 10 units. What is the width of the rectangle?

   1. Write an equation for the perimeter of the rectangle in terms of $x$.
   2. Solve the equation to find the value of $x$.
   3. Find the width of the rectangle using the value of $x$.
   4. Check your answer.

Practice and Problem Solving

10. Error Analysis Describe and correct the error in solving the equation $–2(5 - n) = 2$.

   $–2(5 - n) = 2$
   $–10 - 2n = 2$
   $–10 - 2n + 10 = 2 + 10$
   $–2n = 12$
   $n = -6$

Solve the equation. Check your solution.

11. $13t - 7 - 10t = 2$
12. $22 + 4y - 14 = 0$
13. $2d + 24 + 3d = 84$
14. $4(x + 5) = 16$
15. $3(7 - 2y) = 9$
16. $–2(z + 11) = 6$
17. $–5(3n + 5) = 20$
18. $–30 = 6(f - 5)$
19. $12 = 3(m - 17)$
20. Fishing A family of five people has $200 to spend on fishing rods and fishing licenses. They spend a total of $20 on licenses. Assuming they buy 5 identical rods, what is the maximum amount they can spend on each rod?
21. **Karaoke** You want to organize a group of friends to go to a karaoke studio this Friday night. You must pay $30 to reserve a private karaoke room plus $5 for each person in the group. You also want to have snacks for the group at a cost of $2 per person. How many people can be in the group if you have $70 to spend? 

**Solve the equation. Check your solution.**

22. \(-5(2w + 1) = 25\) 
23. \(4(5 - p) = 8\) 
24. \(-40 - (2x + 5) = -61\) 
25. \(2 = 4(3k - 8) - 11k\) 
26. \(42 = 18t + 4(t + 5)\) 
27. \(-3(2z - 8) + 10z = 16\) 
28. \(-5g - (8 - g) = 12\) 
29. \(-5 = 0.25(4 + 20r) - 8r\) 
30. \(2m + 0.5(m - 4) = 9\) 
31. \(-12 = -2h + 0.2(20 - 6h)\)

32. **Photograph** The perimeter of a rectangular photograph is 22 inches. The length of the photograph is 1 inch more than the width. What are the dimensions of the photograph?

**Geometry** Find the value of \(x\) for the given triangle, rectangle, or square.

33. Perimeter = 40 units 
34. Perimeter = 22 units 

35. Perimeter = 104 units 
36. Perimeter = 32 units 

37. **Cell Phones** Your cell phone provider charges a monthly fee of $19.50 for 200 minutes. You are also charged $.25 per minute for each minute over 200 minutes. Last month, your bill was $29.50.

**a.** Let \(m\) represent the total number of minutes you used last month. Use the verbal model below to write an equation.

<table>
<thead>
<tr>
<th>Total phone bill</th>
<th>Monthly fee</th>
<th>Charge for each additional minute</th>
<th>Number of minutes over 200</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**b.** Solve the equation you wrote in part (a).

**c.** How many additional minutes did you use last month?

38. **Duathlons** A duathlon is an event that consists of running and biking. While training for a duathlon, you run and bike a total of 23 kilometers in 1.25 hours. You run at an average speed of 10 kilometers per hour and bike at an average speed of 24 kilometers per hour. Write and solve an equation to find the time you spend running and the time you spend biking.
39. **Challenge** The figure shown is composed of a triangle and a rectangle. The figure has a total area of 1258 square units. Find the value of $x$.

**Mixed Review**

Plot the point in a coordinate plane. Describe the location of the point. *(Lesson 1.8)*

40. $J(-3, 8)$  
41. $K(8, -3)$  
42. $L(4, -4)$  
43. $M(-1, -1)$

44. $N(0, 2)$  
45. $P(1, 3)$  
46. $Q(-9, 0)$  
47. $R(-5, -8)$

Simplify the expression. *(Lesson 2.3)*

48. $a - 2 - (3 + a)$  
49. $3b + 8 + 2(b - 4)$

50. $-2x + 5 - 7(x + 1)$  
51. $2y - 4 + 3(y + 1)$

52. $-(2x + 3) + 4(x + 2)$  
53. $3(2x - 7) + 8(4 - x)$

54. **Family Party** A family wants to hold a dinner party at a restaurant. The restaurant charges $150 to rent space for the party. The food cost for each person at the party is $18. How many people can come to the party if the family has $600 to spend? *(Lesson 3.1)*

**Standardized Test Practice**

55. **Multiple Choice** What is the solution of the equation $-3(2x - 1) = -21$?

A. $-4$  
B. $-3$  
C. $3$  
D. $4$

56. **Short Response** The length of a rectangle is 5 feet less than twice its width. The perimeter of the rectangle is 38 feet. Let $w$ represent the width. Write an equation for the perimeter of the rectangle in terms of $w$. Then solve the equation to find the length and width of the rectangle.

**Patent Puzzle**

Solve each equation. In each group, there are two equations that have the same solution. Write the value of this solution in the corresponding letter's blank to find the year blue jeans were patented.

A. $10x + 7 = 17$  
$2(7x + 6) = 40$  
$-(x - 11) = 10$

B. $8x - 15 = -47$  
$6(2x - 1) = 90$  
$-7x + 4x = -24$

C. $-5x + 4x = -6$  
$7x - (-12) = 61$  
$7(x + 2) = 63$

D. $2(6x + 7) = 50$  
$-5x - 3x = -56$  
$-11x - 9 = -42$
Solving a Problem Arithmetically and Algebraically

**Goal**
Solve the same problem arithmetically and algebraically.

### Example 1
**Solving a Problem Arithmetically**

The perimeter of the rectangle is 48 centimeters. Find its length \( l \).

**Solution**

The perimeter is the sum of twice the length and twice the width.

1. Multiply the width by 2 to find twice the width: \( 2 \times 6 = 12 \).
2. Subtract twice the width from the perimeter: \( 48 - 12 = 36 \).
3. The difference found in Step 2 represents twice the length, so divide this difference by 2 to find the length: \( \frac{36}{2} = 18 \).

**Answer** The length is 18 centimeters.

### Example 2
**Solving a Problem Algebraically**

The perimeter of the rectangle is 48 centimeters. Find its length \( l \).

**Solution**

\[
P = 2l + 2w \\
48 = 2l + 2(6) \\
48 = 2l + 12 \\
48 - 12 = 2l + 12 - 12 \\
36 = 2l \\
\frac{36}{2} = \frac{2l}{2} \\
18 = l
\]

**Answer** The length is 18 centimeters.
**Example 3**  
*Solving a Problem Arithmetically*

If you run 1 mile per hour faster than your current speed, you can finish the last 3 miles of a race in 0.5 hour. What is your current speed?

**Solution**

1. Divide 3 miles by 0.5 hour to find your new speed: \( 3 \div 0.5 = 6 \)
2. Subtract 1 from your new speed to find your current speed: \( 6 - 1 = 5 \)

**Answer** Your current speed is 5 miles per hour.

**Example 4**  
*Solving a Problem Algebraically*

If you run 1 mile per hour faster than your current speed, you can finish the last 3 miles of a race in 0.5 hour. What is your current speed?

**Solution**

Let \( x \) be your current speed. Then \( x + 1 \) is your new speed.

\[
\begin{align*}
  d &= rt \\
  3 &= (x + 1)0.5 \\
  3 &= 0.5x + 0.5 \\
  2.5 &= 0.5x \\
  5 &= x
\end{align*}
\]

**Answer** Your current speed is 5 miles per hour.

**Practice**

1. **SAVINGS** You have $2800 in your savings account. You want to increase your savings to $4000. You plan to save $20 per week until you reach your goal. How long will it take you to reach your goal?
   - a. Solve the problem arithmetically and algebraically. List your steps.
   - b. Are the operations performed in the same order? Explain.

2. **WORK AREA** You push a table and a desk together to increase your work area. The total work area is now 24 square feet. Find the length \( l \) of the desk.
   - a. Solve the problem arithmetically and algebraically. List your steps.
   - b. Are the operations performed in the same order? Explain.
3.3 Modeling Equations with Variables on Both Sides

**Goal**
Solve equations using algebra tiles.

**Materials**
- algebra tiles

**Investigate**
Use algebra tiles to solve \(4x + 6 = 10 + 2x\).

1. Model \(4x + 6 = 10 + 2x\) using algebra tiles.

2. Remove two \(x\)-tiles from each side.

3. Remove six 1-tiles from each side.

4. Divide the remaining tiles into two equal groups. Each \(x\)-tile is equal to two 1-tiles. So, the solution is 2.

**Draw Conclusions**
Use algebra tiles to model and solve the equation.

1. \(9 + 2x = 1 + 3x\)
2. \(3x + 4 = 8 + x\)
3. \(5x + 2 = 3x + 14\)

4. **Critical Thinking** In the activity above, would you find the correct solution if you performed Step 3 before Step 2? Explain.

5. **Writing** Explain how solving an equation with variables on both sides of the equal sign is different than solving an equation with the variable on one side.
Spanish Club  The Spanish club is arranging a trip to a Mexican restaurant in a nearby city. Those who go must share the $60 cost of using a school bus for the trip. The restaurant’s buffet costs $5 per person. How many students must sign up for this trip in order to limit the cost to $10 per student? In Example 2, you will see how to use an equation to answer this question.

Every equation in this lesson has variables on both sides of the equation. You can solve such an equation by getting the variable terms on one side of the equation and the constant terms on the other side.

Example 1  Solving an Equation with the Variable on Both Sides

Solve \(7n - 5 = 10n + 13\).

\[
\begin{align*}
7n - 5 &= 10n + 13 \\
7n - 5 - 7n &= 10n + 13 - 7n \\
-5 &= 3n + 13 \\
-5 - 13 &= 3n + 13 - 13 \\
-18 &= 3n \\
\frac{-18}{3} &= \frac{3n}{3} \\
-6 &= n
\end{align*}
\]

Answer  The solution is \(-6\).

Checkpoint  Solve the equation. Check your solution.

1. \(5n - 2 = 3n + 6\)  
2. \(8y + 4 = 11y - 17\)  
3. \(m - 1 = 9m + 15\)
How many students must go on the Spanish club trip to the Mexican restaurant, as described on page 131, in order for the cost per student to be $10?

**Solution**

Let $s$ represent the number of students. Write a verbal model.

$$10s = 5s + 60$$

Subtract $5s$ from each side.

$$5s = 60$$

Divide each side by 5.

$$s = 12$$

**Answer** The club needs 12 students to go on the trip.

**Number of Solutions** When you solve an equation, you may find that it has no solution or that every number is a solution.

**Example 3**

**An Equation with No Solution**

Solve $5(2x + 1) = 10x$.

$$5(2x + 1) = 10x$$

Write original equation.

$$10x + 5 = 10x$$

Distributive property

Notice that $10x + 5 = 10x$ is not true because the number $10x$ cannot be equal to 5 more than itself. The equation has no solution. As a check, you can continue solving the equation.

$$10x + 5 - 10x = 10x - 10x$$

Subtract $10x$ from each side.

$$5 = 0$$

Simplify.

The statement $5 = 0$ is not true, so the equation has no solution.

**Example 4**

**Solving an Equation with All Numbers as Solutions**

Solve $6x + 2 = 2(3x + 1)$.

$$6x + 2 = 2(3x + 1)$$

Write original equation.

$$6x + 2 = 6x + 2$$

Distributive property

Notice that for all values of $x$, the statement $6x + 2 = 6x + 2$ is true. The equation has every number as a solution.
**Example 5**  
*Solving an Equation to Find a Perimeter*

**Geometry** Find the perimeter of the square.

1. A square has four sides of equal length. Write an equation and solve for \( x \).

\[
2x = x + 4 \\ 2x - x = x + 4 - x \quad \text{Write equation.} \\ x = 4 \quad \text{Subtract } x \text{ from each side.} \\
\]

2. Find the length of one side by substituting 4 for \( x \) in either expression.

\[
2x = 2(4) = 8 \quad \text{Substitute 4 for } x \text{ and multiply.}
\]

3. To find the perimeter, multiply the length of one side by 4.

\[4 \cdot 8 = 32\]

**Answer** The perimeter of the square is 32 units.

---

**Guided Practice**

**Vocabulary Check**

1. Describe what steps you would take to solve \( 8x + 5 = 2x - 7 \).
2. Explain why the equation \( 5z + 2 = 5z \) has no solution.

**Skill Check**  
*Solve the equation. Check your solution.*

3. \( 13m - 22 = 9m - 6 \)
4. \( 19c + 26 = 41 + 14c \)
5. \( 15 - 4x = 42 - 7x \)
6. \( 14 + 5y = 50 - 4y \)
7. \( 18w - 2 = 10w + 14 \)
8. \( -5a + 6 = 6a - 38 \)
9. **Error Analysis** Describe and correct the error in solving the equation \( 4x + 7 = x - 2 \).

**Shopping** You spend $60 on clothes and buy 3 DVD movies. Your friend spends nothing on clothes and buys 8 DVD movies. You both spend the same amount of money. All the DVDs cost the same amount. How much does each DVD cost?
Solve the equation. Check your solution.

11. \(25u + 74 = 23u + 92\)
12. \(-5k - 19 = 5 - 13k\)
13. \(-11y + 32 = 104 - 5y\)
14. \(-15n + 16 = 86 - 29n\)
15. \(25t = 5(5t + 1)\)
16. \(13 - 3p = -5(3 + 2p)\)
17. \(-24s - 53 = 39 - s\)
18. \(14a - 93 = 49 - 57a\)
19. \(7(2p + 1) = 14p + 7\)
20. \(8v = 2(4v + 2)\)
21. \(3x + 6 = 3(2 + x)\)
22. \(2(-4h - 13) = 37 + 13h\)

Write the verbal sentence as an equation. Then solve the equation.

23. Nine plus 2 times a number is equal to 2 less than 3 times the number.
24. Three less than 11 times a number is equal to 9 plus 5 times the number.
25. Four minus 7 times a number is equal to 12 minus 3 times the number.
26. Twelve less than \(-9\) times a number is equal to 8 minus 4 times the number.

27. **Toll Booth** You lose your electronic tag that you use to pay tolls on the highway in your city. It costs you $24 to replace the tag. The cost of one toll when you don’t use the tag is $3. The cost of the same toll when you do use the tag is $1.50. How many times will you have to use the tag to pay for the tolls in order for the total cost to be the same as not using the tag?

Find the perimeter of the square.

28. \(36 - 5x\)
29. \(12x\)
30. \(5x + 32\)

31. **Driving** A family is driving to Houston, Texas. A sign indicates that they are 700 miles from Houston. Their car’s trip odometer indicates that they are 400 miles from home. They are traveling at an average speed of 60 miles per hour.
   a. Write an expression for the distance (in miles) they will be from Houston in \(x\) hours.
   b. Write an expression for the distance (in miles) they will be from home in \(x\) hours.
   c. Use the expressions from parts (a) and (b) to write and solve an equation to find the number of hours they will drive until they are exactly halfway between Houston and their home.
   d. Suppose they travel by local roads instead of the highway. They travel the 700 miles at a speed of 45 miles per hour. How long will they drive before they are exactly halfway between Houston and their home?
32. **Pasta Machine** A pasta machine costs $33. The ingredients to make one batch of pasta cost $.33. The same amount of pasta purchased at a store costs $.99. How many batches of pasta will you have to make for the cost of the machine and ingredients to equal the cost of buying the same amount of pasta at the store?

33. **Logical Reasoning** Copy and complete the steps for solving the equation shown. Use properties to justify as many steps as possible.

\[ 5t - 24 = 8t \]
\[ 5t - 24 - 5t = 8t - 5t \]
\[ -24 = 3t \]
\[ \frac{-24}{3} = \frac{3t}{3} \]
\[ -8 = t \]

\[ 5t - 24 - 5t = 8t - 5t \]
\[ b. ? \]

\[ \frac{-24}{3} = \frac{3t}{3} \]
\[ c. ? \]

\[ -8 = t \]
\[ d. ? \]

\[ e. ? \]

**Solve the equation. Check your solution.**

34. \( 3x - 7 = 8 + 6(x + 2) \)

35. \( 13y + 19 = 6(9 + y) + 14 \)

36. \( 8(z + 4) = 5(13 + z) \)

37. \( 8a - 2(a + 5) = 2(a - 1) \)

38. **Geometry** The perimeter of the square is equal to the perimeter of the triangle. The sides of the triangle are equal in length.

a. **Estimate** Without doing any calculations, estimate which figure has the greater side length. Explain your choice.

b. What is the side length and perimeter of each figure?

**Use a calculator to solve the equation. Check your solution.**

39. \( 0.75m + 14 = 1.87m - 10.3936 \)

40. \( 19.5 + 0.5t = 10.6206 - 0.4t \)

41. \( -9.39 - 3.4d = -1.1d + 11.08 \)

42. \( -130.5 - 9b = -55.104 + 3.2b \)

43. **Challenge** Consider the equation \( ax + 6 = 2(x + 3) \). For what value(s) of \( a \) does the equation have all numbers as a solution? For what value(s) of \( a \) does the equation have just one solution?

Mixed Review

**Algebra Basics** Solve the equation. Check your solution. (Lesson 2.5)

44. \( c - 20 = 14 \)

45. \( d + 9 = -12 \)

46. \( x - 3 = 17 \)

47. \( y - 21 = -15 \)

48. **Gym** To join a gym, you pay a one-time fee of $75 and $45 per month for the duration of the membership. You have paid a total of $345. How long have you been a member of the gym? (Lesson 3.1)

49. The perimeter of the square shown is 32 units. Find the value of \( x \). (Lesson 3.2)

**Standardized Test Practice**

50. **Multiple Choice** What is the solution of the equation \( 2(3x + 4) = 6x + 5? \)
   
   | A. 1 | B. 3 | C. All numbers | D. No solution |

51. **Multiple Choice** For which equation is 6 a solution?
   
   F. \( -2y - 7 = 11 - 5y \)
   
   G. \( 11y - 32 = 7y - 12 \)
   
   H. \( 18y - 16 = 13y + 19 \)
   
   I. \( -7y - 24 = -8 - 9y \)
3.3 Solving Equations

**Goal** Use a table to solve an equation with the variable on both sides.

**Example**

**Use a table to solve** \(5x - 1 = 4x + 3\).

1. Enter the expressions on each side of the equal sign into a graphing calculator. The expression on the left is called Y1, and the expression on the right is Y2.

   **Keystrokes**
   
   \[
   \begin{align*}
   &Y1 = 5x - 1 \\
   &Y2 = 4x + 3 \\
   &Y3 = x \\
   &Y4 = 3 \\
   \end{align*}
   \]

2. Use the calculator’s table feature to find the value of each expression for different values of \(x\). Press \(\text{[2nd]} \text{[TBLSET]}\) and enter the settings shown on the first screen below. (\(\Delta\text{Tbl}\) represents the increment the calculator uses to go from one \(x\)-value to the next in the table.) Then, press \(\text{[2nd]} \text{[TABLE]}\) to display the table shown on the second screen.

3. Compare the values of the expression in the Y1 column with the values of the expression in the Y2 column. The values are the same when \(x = 4\). So, the solution of the equation \(5x - 1 = 4x + 3\) is 4.

**Draw Conclusions**

**Use a table to solve the equation.**

1. \(x - 2 = 2x - 6\)  
2. \(3x + 1 = x + 7\)  
3. \(12 - x = x - 4\)  
4. \(7x = 16 - x\)  
5. \(5x + 2 = 8x - 1\)  
6. \(4x - 6 = 2x + 4\)  
7. **Critical Thinking** Solve the equation \(3x + 6 = 13x + 2\) using paper and a pencil. Explain how you would change the settings in the TABLE SETUP menu so that you could solve the equation using a calculator.
Rewriting Equations and Formulas

**GOAL** Rewrite literal equations and formulas.

A **literal equation**, such as \( ax + b = c \), is an equation in which the coefficients and constants have been replaced by letters. The equations \( 2x + 1 = 5 \) and \( 3x + 4 = 19 \) have the general form \( ax + b = c \). When you solve a literal equation, you can use the result to solve any equation that has the same form as the literal equation.

### Example 1  
**Solving a Literal Equation**  

Solve \( ax + b = c \) for \( x \). Then use the solution to solve \( 3x + 4 = 19 \).

**Solution**

1. Solve \( ax + b = c \) for \( x \).
   
   \[
   ax + b = c \\
   ax = c - b \\
   x = \frac{c - b}{a}
   \]
   
   Write original equation.
   
   Subtract \( b \) from each side.
   
   Assume \( a \neq 0 \). Divide each side by \( a \).

2. Use the solution to solve \( 3x + 4 = 19 \).
   
   \[
   x = \frac{c - b}{a} \\
   = \frac{19 - 4}{3} \\
   = 5
   \]
   
   Solution of literal equation
   
   Substitute 3 for \( a \), 4 for \( b \), and 19 for \( c \).
   
   Simplify.

**Answer** The solution of \( 3x + 4 = 19 \) is 5.

### Example 2  
**Rewriting an Equation**  

Solve \( 4x + 2y = 16 \) for \( y \).

\[
4x + 2y = 16 \\
2y = 16 - 4x \\
\frac{1}{2}(2y) = \frac{1}{2}(16 - 4x) \\
y = 8 - 2x
\]

Write original equation.

Subtract \( 4x \) from each side.

Multiply each side by \( \frac{1}{2} \).

Use distributive property, then simplify.
Example 3  

Rewriting and Using a Geometric Formula

The perimeter $P$ of a rectangle is given by the formula $P = 2l + 2w$ where $l$ is the length and $w$ is the width.

a. Solve the formula for the length $l$.

b. Use the rewritten formula to find the length of the rectangle shown.

Solution

a. $P = 2l + 2w$  
   Write original formula.

   $P - 2w = 2l$  
   Subtract $2w$ from each side.

   $\frac{1}{2}(P - 2w) = \frac{1}{2}(2l)$  
   Multiply each side by $\frac{1}{2}$.

   $\frac{1}{2}P - w = l$  
   Use distributive property, then simplify.

b. Substitute $96$ for $P$ and $18$ for $w$ in the rewritten formula.

   $l = \frac{1}{2}P - w$  
   Write rewritten formula.

   $= \frac{1}{2}(96) - 18$  
   Substitute $96$ for $P$ and $18$ for $w$.

   $= 30$  
   Simplify.

Answer  The length of the rectangle is 30 centimeters.

Practice

Solve the literal equation for $x$. Then use the solution to solve the specific equation.

1. $a + bx = c; 8 + 3x = 2$  
2. $ax - b = c; 5x - 4 = 16$

3. $ax = bx - c; 3x = 2x - 6$  
4. $a(x + b) = c; 2(x + 8) = 36$

5. $\frac{x}{a} = b; \frac{x}{9} = 7$  
6. $\frac{x}{a} + b = c; \frac{x}{6} + 5 = 4$

Solve the equation for $y$.

7. $4x + y = 11$  
8. $y - 13x = 9$  
9. $2x - 2y = 10$

10. $3x + 6y = 12$  
11. $15 = 5x + 10y$  
12. $7y + 14 = 21x$

13. $2x - 3y = 6$  
14. $24 = 3x + 4y$  
15. $2 + 3y = 5 - 9x$

16. Geometry  The area of a triangle is given by the formula $A = \frac{1}{2}bh$ where $b$ is the base and $h$ is the height.

a. Solve the formula for the base $b$.

b. Use the rewritten formula to find the base of the triangle shown, which has an area of 135 square yards.
Write the verbal sentence as an equation. Then solve the equation.

1. Twice a number plus 5 is equal to 27.
2. Seven times the sum of 4 and a number is −14.
3. Three more than 4 times a number is equal to 9 less than twice the number.

Solve the equation. Check your solution.

4. \(11k + 9 = 42\)
5. \(\frac{a}{3} + 11 = -5\)
6. \(\frac{w}{2} - 18 = -7\)
7. \(2 + 5t - 3 = 34\)
8. \(-3y + 15 - y = 39\)
9. \(5(n + 2) = 10\)
10. \(2 - 5(h + 3) = -28\)
11. \(5s = 7s + 1 - 2s\)
12. \(4d - 5 = -d\)
13. \(17 - 5m = 50 + 6m\)
14. \(3f - 12 = 3(f - 12)\)
15. \(8(4p + 1) = 32p + 8\)

16. **Income**  
Your friend works as a waitress at a local restaurant. Her income consists of an hourly wage plus tips. On Wednesday, your friend earned $25 in tips over a 5 hour period. On Friday, your friend earned $30.76 in tips over a 3 hour period. How much is your friend’s hourly wage if your friend earned the same amount of money on Wednesday as on Friday?

17. All three sides of the triangle shown are equal in length. Find the perimeter of the triangle.

**Brain Game**

Two people are packing equal numbers of small boxes into large boxes. One person has 3 large boxes that are full of smaller boxes and 24 small boxes that are not yet packed. The other person has 5 large boxes that are full of smaller boxes and 10 small boxes that are not yet packed. Each large box holds the same number of small boxes. How many small boxes can each large box hold? What is the total number of small boxes each person will pack? How many large boxes will each person need in order to pack all of his or her small boxes?
Solving Inequalities Using Addition or Subtraction

An **inequality** is a statement formed by placing an inequality symbol between two expressions. For example, \( y + 5 \leq -6 \) is an inequality.

The **solution of an inequality** with a variable is the set of all numbers that produce true statements when substituted for the variable. You can show the solution of an inequality by graphing the inequality on a number line. When you graph an inequality of the form \( x > a \) or \( x < a \), use an open circle at \( a \). When you graph an inequality of the form \( x \geq a \) or \( x \leq a \), use a closed circle at \( a \).

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Words</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x &lt; 3 )</td>
<td>All numbers less than 3</td>
<td><img src="https://example.com/graph1" alt="Graph" /></td>
</tr>
<tr>
<td>( y &gt; 2 )</td>
<td>All numbers greater than 2</td>
<td><img src="https://example.com/graph2" alt="Graph" /></td>
</tr>
<tr>
<td>( z \leq 4 )</td>
<td>All numbers less than or equal to 4</td>
<td><img src="https://example.com/graph3" alt="Graph" /></td>
</tr>
<tr>
<td>( n \geq 2 )</td>
<td>All numbers greater than or equal to 2</td>
<td><img src="https://example.com/graph4" alt="Graph" /></td>
</tr>
</tbody>
</table>

**Example 1** Writing and Graphing an Inequality

**Science** The freezing point of water is 0°C. At temperatures at or below the freezing point, water is a solid (ice). Write an inequality that gives the temperatures at which water is a solid. Then graph the inequality.

**Solution**

Let \( t \) represent the temperature of water. Water is a solid at temperatures less than or equal to 0°C.

**Answer** The inequality is \( t \leq 0 \). The graph is shown below.
Solving Inequalities

You can use the following properties to find the solutions of inequalities involving addition and subtraction. Using these properties, you can write equivalent inequalities. Equivalent inequalities are inequalities that have the same solution.

**Example 2**  
**Solving an Inequality Using Subtraction**

Solve \( m + 5 \geq 10 \). Graph and check your solution.

\[
\begin{align*}
\quad m + 5 & \geq 10 \quad \text{Write original inequality.} \\
\quad m + 5 - 5 & \geq 10 - 5 \quad \text{Subtract 5 from each side.} \\
\quad m & \geq 5 \quad \text{Simplify.}
\end{align*}
\]

**Answer** The solution is \( m \geq 5 \).

✓ **Check** Choose any number greater than or equal to 5. Substitute the number into the original inequality.

\[
\begin{align*}
\quad m + 5 & \geq 10 \quad \text{Write original inequality.} \\
\quad 8 + 5 & \geq 10 \quad \text{Substitute 8 for } m. \\
\quad 13 & \geq 10 \quad \text{Solution checks.}
\end{align*}
\]

**Example 3**  
**Solving an Inequality Using Addition**

Solve \( -10 > x - 12 \). Graph your solution.

\[
\begin{align*}
\quad -10 & > x - 12 \quad \text{Write original inequality.} \\
\quad -10 + 12 & > x - 12 + 12 \quad \text{Add 12 to each side.} \\
\quad 2 & > x \quad \text{Simplify.}
\end{align*}
\]

**Answer** The solution is \( 2 > x \), or \( x < 2 \).

✓ **Checkpoint**

Solve the inequality. Graph and check your solution.

1. \( n + 7 > 3 \)  
2. \( 10 \geq y + 4 \)  
3. \( -6 \leq x - 9 \)  
4. \( z - 5 < 1 \)
Example 4  Writing and Solving an Inequality

**Triathlon** You are competing in a triathlon, a sports competition with three events. Last year, you finished the triathlon in 85 minutes. The table shows your times for this year’s first two events. What possible times can you post in the running event and still beat last year’s finishing time?

<table>
<thead>
<tr>
<th>Event</th>
<th>Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swimming</td>
<td>17</td>
</tr>
<tr>
<td>Biking</td>
<td>45</td>
</tr>
<tr>
<td>Running</td>
<td>?</td>
</tr>
</tbody>
</table>

**Solution**

Let \( t \) represent this year’s running time. Write a verbal model.

\[
17 + 45 + t < 85 \quad \text{Substitute.}
\]
\[
62 + t < 85 \quad \text{Simplify.}
\]
\[
62 + t - 62 < 85 - 62 \quad \text{Subtract 62 from each side.}
\]
\[
t < 23 \quad \text{Simplify.}
\]

**Answer** To beat last year’s finishing time, you must post a time in the running event that is less than 23 minutes.

### 3.4 Exercises

**Guided Practice**

**Vocabulary Check**

1. What are equivalent inequalities?
2. Explain how the graph of \( x > 5 \) is different from the graph of \( x \geq 5 \).

**Skill Check**

Tell whether the given number is a solution of \(-5 < n\).

3. 8  4. \(-8\)  5. \(-4\)  6. 4

Solve the inequality. Graph and check your solution.

7. \( x + 2 > -3 \)  8. \( 1 \geq x - 9 \)  9. \( x + 4 < 3 \)  10. \( x + 3 > 7 \)

**Guided Problem Solving**

11. **Astronauts** To become a NASA pilot astronaut, a NASA pilot must log at least 1000 hours as pilot-in-command of a jet aircraft. A NASA pilot has completed all other qualifications and has 250 hours logged. How many more hours must the pilot log to become a pilot astronaut?

   1. Write an inequality to represent the situation.
   2. Solve the inequality. Then graph and check the solution.
   3. Interpret the solution in terms of the real-life situation.
Write an inequality to represent the situation.

12. The greatest weight that a forklift can raise is 2500 pounds.

13. The speed limit is 55 miles per hour.

14. A truck can tow a maximum weight of 7700 pounds.

15. You must be at least 48 inches tall to ride the roller coaster.

16. You can save up to $50 on DVD players this week.

Write an inequality represented by the graph.

17. \[ \begin{align*} \text{12} &< x < 5 \\ \end{align*} \]

18. \[ \begin{align*} -10 &< y < 7 \\ \end{align*} \]

Solve the inequality. Graph your solution.

21. \[ x + 4 < 5 \]

22. \[ m + 8 \geq 12 \]

23. \[ -11 < y + 5 \]

24. \[ -8 \geq d - 7 \]

25. \[ -45 > g - 16 \]

26. \[ z - 15 > 72 \]

27. \[ f + 1 \geq -8 \]

28. \[ h + 19 \leq 15 \]

29. \[ 18.1 \leq p - 7 \]

30. \[ t - 7 < 3.4 \]

31. \[ b + 2.5 \leq 2.5 \]

32. \[ a - 10.2 > 5.3 \]

33. **Neon** The lowest temperature at which neon is a gas, called its boiling point, is \(-411^\circ F\). Write and graph an inequality to show the temperatures at which neon is a gas.

Solve the inequality. Graph your solution.

34. \[ 5 + m + 8 \geq 14 \]

35. \[ 13 + n - 26 < 38 \]

36. \[ 2.35 + p + 14.9 > 49.25 \]

37. \[ q + 4 + 16 \geq 30 \]

38. **Bacteria** In 1969, Apollo 12 astronauts found a small colony of *Streptococcus* bacteria that had apparently traveled unprotected to the moon on the Surveyor 3 spacecraft’s TV camera about three years earlier. The bacteria survived at temperatures as low as \(-280^\circ F\). Write and graph an inequality to show the temperatures at which the bacteria survived.

39. **Writing** Is it possible to check all the numbers that are solutions of an inequality? Explain. Does checking just one number guarantee that a solution is correct?

40. **Train Travel** You are traveling by train. You are allowed two carryon bags, each with a maximum weight of 50 pounds. You have two bags: one that weighs 14 pounds and one that weighs 21 pounds.

   a. Write and solve an inequality that represents the weight \( w \) (in pounds) of personal belongings you can add to the first bag without exceeding the weight limit.

   b. Write and solve an inequality that represents the weight \( w \) (in pounds) of personal belongings you can add to the second bag without exceeding the weight limit.
In Exercises 41 and 42, graph the compound inequality. A compound inequality consists of two inequalities joined by the word and or or.

**Example**

**Graphing Compound Inequalities**

Graph the compound inequality \( x > 3 \) and \( x < 10 \).

Include numbers that are both greater than 3 and less than 10.

41. \( x \geq -1 \) and \( x \leq 4 \)  
42. \( x < 3 \) and \( x \geq 0 \)

43. **Skiing** The ski wax you use keeps your skis performing well at temperatures from \(-6^\circ C\) to \(15^\circ C\). Express the lower limit of the ski wax as an inequality, and express the upper limit as an inequality. Then write the inequalities as a compound inequality and graph it.

44. **Challenge** Explain how you can graph the compound inequality \( x \leq 8 \) or \( x \geq 10 \). How does this graph look different from the graph of \( x \geq 8 \) and \( x \leq 10 \)?

**Mixed Review**

45. **Geometry** Find the length of a side of a square with a perimeter of 36.6 meters. (Lesson 2.7)

46. **Fundraising** A basketball team is raising money for uniforms and equipment. So far, the team has raised $1275. The team plans to spend $450 on equipment and buy as many uniforms as possible. Each uniform costs $55. How many uniforms can the team buy with the money it has raised? (Lesson 3.1)

Write the verbal sentence as an equation. Then solve the equation. (Lesson 3.3)

47. Five plus 4 times a number is equal to the sum of 7 times the number and 11.

48. Eight less than 3 times a number is equal to \(-3\) plus twice the number.

**Standardized Test Practice**

49. **Multiple Choice** Which inequality is represented by the graph shown?

A. \( y < -2 \)  
B. \( y > -2 \)  
C. \( y \leq -2 \)  
D. \( y \geq -2 \)

50. **Multiple Choice** Which number is a solution of the inequality \( b + 2 > 2 \)?

F. 4  
G. 0  
H. -1  
I. -2
3.5 Multiplication and Division Properties of Inequality

Determine how multiplication or division affects an inequality.

1. Choose two different integers and insert an inequality symbol between them to make a true statement.

   \(-2 < 4\)

2. Multiply each number in the original inequality by 2. Is the new inequality a true statement?

   \(2 \cdot (-2) \leq 2 \cdot 4\)
   \(-4 \leq 8 \checkmark\)
   Yes, \(-4\) is less than 8.

3. Multiply each number in the original inequality by \(-2\). Is the new inequality a true statement?

   \(-2 \cdot (-2) \geq -2 \cdot 4\)
   \(4 < -8 \times\)
   No, 4 is not less than \(-8\).

4. Divide each number in the original inequality by 2. Is the new inequality a true statement?

   \(\frac{-2}{2} \leq \frac{4}{2}\)
   \(-1 < 2 \checkmark\)
   Yes, \(-1\) is less than 2.

5. Divide each number in the original inequality by \(-2\). Is the new inequality a true statement?

   \(\frac{-2}{-2} \leq \frac{4}{-2}\)
   \(1 < -2 \times\)
   No, 1 is not less than \(-2\).

Draw Conclusions

1. Critical Thinking Repeat the steps above with a new pair of integers. In Steps 3 and 5, what could you do to the inequality symbols to make the statements true?

Given that \(a > b\), copy and complete using \(<\) or \(\geq\) to make a true statement.

1. \(\frac{a}{2} \geq \frac{b}{2}\)
2. \(\frac{a}{-2} \geq \frac{b}{-2}\)
3. \(-a \geq -b\)
4. \(3a \geq 3b\)
Geese Migration Some flocks of Canada geese can fly nonstop for up to 16 hours. In this time, a flock can migrate as far as 848 miles. At what average speeds can such a flock fly during migration? In Example 3, you will see how to answer this question by solving an inequality.

As shown below, when each side of the inequality $2 < 8$ is multiplied by a positive number, the inequality remains true. When each side is multiplied by a negative number, the inequality sign must be reversed.

\[
\begin{align*}
2 < 8 & \quad 2 \cdot 4 < 8 \cdot 4 \\
8 < 32 & \quad -8 > -32
\end{align*}
\]

Reverse inequality sign.

These examples suggest the following rules for solving inequalities.

**Multiplication Property of Inequality**

**Words** Multiplying each side of an inequality by a positive number produces an equivalent inequality.

Multiplying each side of an inequality by a negative number and reversing the direction of the inequality symbol produces an equivalent inequality.

**Algebra** If $a < b$ and $c > 0$, then $ac < bc$.

If $a < b$ and $c < 0$, then $ac > bc$.

**Example 1** Solving an Inequality Using Multiplication

\[
\begin{align*}
\frac{m}{-3} & > 3 \\
-3 \cdot \frac{m}{-3} & < -3 \cdot 3 \\
m & < -9
\end{align*}
\]

Original inequality

Multiply each side by $-3$.

Reverse inequality symbol.

Simplify.
Find the average speeds at which the flock of Canada geese described on page 146 can fly during migration.

**Solution**

Let $s$ represent the average flight speeds. Write a verbal model.

<table>
<thead>
<tr>
<th>Flight time</th>
<th>Average flight speeds</th>
<th>Maximum flight distance</th>
</tr>
</thead>
</table>

\[
16s \leq 848 
\]

Substitute.

\[
\frac{16s}{16} \leq \frac{848}{16}
\]

Divide each side by 16.

\[
s \leq 53
\]

Simplify.

**Answer** The flock of Canada geese can fly at average speeds of 53 miles per hour or less during migration.

**Division** The rules for solving an inequality using division are like the rules for solving an inequality using multiplication.

**Division Property of Inequality**

**Words** Dividing each side of an inequality by a positive number produces an equivalent inequality.

Dividing each side of an inequality by a negative number and reversing the direction of the inequality symbol produces an equivalent inequality.

**Algebra**

If $a < b$ and $c > 0$, then \[ \frac{a}{c} < \frac{b}{c}. \]

If $a < b$ and $c < 0$, then \[ \frac{a}{c} > \frac{b}{c}. \]

**Example 2**

**Solving an Inequality Using Division**

\[
-10t \geq 34
\]

Original inequality

\[
\frac{-10t}{-10} \leq \frac{34}{-10}
\]

Divide each side by $-10$.

Reverse inequality symbol.

\[
t \leq -3.4
\]

Simplify.

**Checkpoint**

Solve the inequality. Graph your solution.

1. \( \frac{n}{6} > 7 \)
2. \( \frac{t}{4} \leq 8 \)
3. \( 2x > -8 \)
4. \( -7s \leq 14 \)

**Example 3**

**Writing and Solving an Inequality**

Find the average speeds at which the flock of Canada geese described on page 146 can fly during migration.

**Solution**

Let $s$ represent the average flight speeds. Write a verbal model.

In the Real World

**Geese Migration** In North America, not all Canada geese migrate. About 3.6 million “resident” geese live in urban and suburban areas, such as parks, throughout the year. Resident Canada geese outnumber migrating Canada geese. Write an inequality that compares the numbers of migrating geese and resident geese.
## Guided Practice

### Vocabulary Check
1. Which property would you use to solve the inequality \(-7y \leq 49\)?
2. Explain how solving \(2x > -14\) is different from solving \(-2x > 14\).

### Skill Check
Solve the inequality. Graph and check your solution.

3. \(\frac{v}{2} < -8\)  
4. \(8b > 32\)  
5. \(\frac{u}{6} \geq 3\)  
6. \(-6s \leq 54\)
7. \(5a < -35\)  
8. \(\frac{p}{7} > 6\)  
9. \(3r \geq 21\)  
10. \(\frac{t}{4} \leq -9\)

### Guided Problem Solving
11. Training While training for a marathon, you try to consume at least 2400 Calories each day. For one meal, you like to eat at least 500 Calories. You choose to eat pasta that has 200 Calories per cup. How many cups of pasta should you eat?

   1) Let \(c\) represent the number of cups of pasta. Write an inequality based on the verbal model given below.
   
<table>
<thead>
<tr>
<th>Calories per cup</th>
<th>Number of cups</th>
<th>Total calories for meal</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>(c)</td>
<td>(200c)</td>
</tr>
<tr>
<td>500</td>
<td>(c)</td>
<td>(500c)</td>
</tr>
</tbody>
</table>
   
   2) Solve the inequality.
   
   3) Explain what the solution tells you about the situation.

## Practice and Problem Solving

Solve the inequality. Graph your solution.

12. \(\frac{a}{2} < -9\)  
13. \(\frac{b}{7} > 7\)  
14. \(\frac{c}{8} \geq 3\)  
15. \(-16y > 48\)
16. \(5z < 65\)  
17. \(\frac{d}{11} \leq 6\)  
18. \(12x \geq -60\)  
19. \(4w \leq 68\)
20. \(\frac{f}{9} < -12\)  
21. \(\frac{h}{-6} \leq 13\)  
22. \(-16k \geq 96\)  
23. \(6q > -84\)
24. \(-7s \geq -84\)  
25. \(4m < -60\)  
26. \(\frac{v}{5} > -2\)  
27. \(\frac{n}{-3} \geq -5\)
28. Error Analysis Describe and correct the error in solving the inequality \(9x > -45\).
29. **In-Line Skates** You want to use in-line skates. You can either rent in-line skates for $12 per day or purchase them for $60. How many times will you have to use the in-line skates in order for the cost of purchasing them to be less than the total cost of renting them?

**Write the verbal sentence as an inequality. Then solve the inequality.**

30. Five times a number is at least 45.

31. A number divided by 4 is at most 8.

32. A number divided by $-3$ is less than 6.

33. Seven times a number is greater than $-35$.

34. A number divided by 2 is no more than 5.

35. Three times a number is more than $-18$.

36. **Extended Problem Solving** The weight limit for freight loaded onto a freight elevator is 7500 pounds. The elevator is being used to move 50 heavy crates. Each crate weighs 375 pounds.

   a. **Interpret** Write and solve an inequality to determine how many crates you can move in one trip on the elevator. Assume that weight is the only factor affecting how many crates you can move at one time.

   b. **Apply** How many times will you need to load the elevator to move all of the crates? Explain.

37. **Reading** You need to read at least 105 pages of a book for your English class in the next 7 days. How many pages should you read each day?

38. **Biking** You want to bike at least 45 miles as part of a training program. If you bike for 5 hours, what average speeds will allow you to meet your goal?

   Use a calculator to solve the inequality. Graph your solution.

39. $-8.9b \geq 40.94$

40. $\frac{x}{2.4} \geq 8.5$

41. $\frac{z}{7.2} < -3.4$

42. $6.3a > 10.71$

43. $-3.9c \leq 43.68$

44. $\frac{y}{-9.1} \leq 6.5$

45. **Water** Filling the bathtub uses 60 gallons of water. Taking a shower uses 2 gallons per minute. How many minutes can you be in the shower and still use less water than you would by filling the bathtub?

46. **Caribou** A herd of caribou can migrate as far as 36 miles in 24 hours.

   a. Write and solve an inequality to find the average speeds (in miles per hour) at which caribou can migrate.

   b. A caribou herd has been moving for three days. On a number line, graph the distances (in miles) the herd could have traveled.

47. **Carpeting** Your parents have decided to install new carpeting in your room, which is rectangular and measures 10 feet by 12 feet. They want to spend at most $200 on the carpeting. At the flooring store, carpeting is sold by the square foot. How much money will your parents spend per square foot for carpeting?
48. **Critical Thinking** The inequalities $2x < 3$ and $4x < 6$ are equivalent inequalities. Write a third inequality equivalent to $2x < 3$ and $4x < 6$.

49. **Challenge** An underwater camera can withstand pressures up to 1500 pounds per square inch. The formula $P = 14.7 + 0.45d$ can be used to find the water pressure $P$ (in pounds per square inch) at depth $d$ (in feet) underwater. Find the depths at which the camera can be used.

**Mixed Review**

**Algebra Basics** Solve the equation. Check your solution. *(Lesson 2.7)*

50. $x + 3.5 = 9.2$  
51. $x - 6.7 = 5.8$  
52. $44.72 = 5.2x$  
53. $\frac{x}{7.6} = 9.5$

54. Find the perimeter of the square. *(Lesson 3.3)*

**Solve the inequality. Graph your solution.** *(Lesson 3.4)*

55. $x + 12 > 96$  
56. $x + 17 \geq 44$  
57. $x - 26 \leq 33$  
58. $x - 14 < 29$

**Standardized Test Practice**

59. **Multiple Choice** Which number is not a solution of $\frac{t}{-9} \geq 3$?
   
   A. $-35$  
   B. $-30$  
   C. $-27$  
   D. $-25$

60. **Multiple Choice** Which number is a solution of $\frac{x}{-7} < 6$?
   
   F. $-100$  
   G. $-56$  
   H. $-42$  
   I. $-14$

61. **Multiple Choice** Which inequality is equivalent to $-18 \leq 3p$?
   
   A. $p \geq -54$  
   B. $-54 \geq p$  
   C. $p \geq -6$  
   D. $-6 \geq p$

**Youthest to Oldest**

Use the given information to list the six cousins in order from least to greatest age and give their ages. No two cousins are the same age.

- Erika is 4 years old.
- Charlie's age is greater than 4 times Dawn's age.
- All the girls' ages are greater than Anthony's age.
- All the cousins' ages are less than or equal to 13.
- Matthew is older than exactly three cousins.
- Stephanie is 6 years older than Erika.
- All the cousins' ages are greater than or equal to 2.
- Erika is 1 year older than Dawn.
- One boy is 6 years old.
Solving Multi-Step Inequalities

**Example 1: Writing and Solving a Multi-Step Inequality**

**Soccer** Your school’s soccer team is trying to break the school record for goals scored in one season. Your team has already scored 88 goals this season. The record is 138 goals. With 10 games remaining on the schedule, how many goals, on average, does your team need to score per game to break the record?

To solve a multi-step inequality like \(2x + 1 > 5\), you should use the properties of inequality from Lessons 3.4 and 3.5 to get the variable terms on one side of the inequality and the constant terms on the other side.

**Solution**

Let \(g\) represent the average number of goals scored per game. Write a verbal model.

\[
\begin{align*}
\text{Goals scored this season} + \text{Number of games left} \cdot \text{Goals scored per game} & > \text{School record} \\
88 + 10g & > 138 \\
88 + 10g - 88 & > 138 - 88 \\
10g & > 50 \\
\frac{10g}{10} & > \frac{50}{10} \\
g & > 5
\end{align*}
\]

**Answer** Your team must score, on average, more than 5 goals per game.

**Checkpoint**

1. Look back at Example 1. Suppose the season goal record is 124 goals and your team has already scored 52 goals. With 12 games remaining on the schedule, how many goals, on average, does your team need to score per game to break the record?
Chapter 3  Multi-Step Equations and Inequalities

### Example 2  Solving a Multi-Step Inequality

\[
\frac{x}{-4} - 6 \geq -5 \\
\frac{x}{-4} - 6 + 6 \geq -5 + 6 \\
\frac{x}{-4} \geq 1 \\
-4 \cdot \frac{x}{-4} \leq -4 \cdot 1 \\
x \leq -4
\]

Original inequality
Add 6 to each side.
Simplify.
Multiply each side by \(-4\).
Reverse inequality symbol.
Simplify.

### Example 3  Combining Like Terms in a Multi-Step Inequality

**Ice Skating** You plan to go ice skating often this winter. The skating rink charges $4 for admission. You can either rent ice skates at the skating rink for $5 per day or buy your own pair for $45. How many times do you have to use the ice skates in order for the cost of buying them to be less than the total cost of renting them?

**Solution**

You have two options: buying skates or renting skates. Let \(v\) represent the number of visits to the skating rink. Write a variable expression for the cost of each option.

**Option 1: Buying Skates**

\[
\text{Cost of skate rental fee} + \text{Number of visits} \cdot \text{Admission fee} = 45 + 4v
\]

**Option 2: Renting Skates**

\[
\text{Skate rental fee} \cdot \text{Number of visits} + \text{Admission fee} \cdot \text{Number of visits} = 5v + 4v, \text{ or } 9v
\]

To find the values of \(v\) for which the cost of option 1 is less than the cost of option 2, write and solve an inequality.

\[
45 + 4v < 9v \\
45 + 4v - 4v < 9v - 4v \\
45 < 5v \\
\frac{45}{5} < \frac{5v}{5} \\
9 < v
\]

**Answer** If you buy skates, the cost will be less after more than 9 visits.
Lesson 3.6  
Solving Multi-Step Inequalities

Guided Practice

Vocabulary Check
1. Write and solve an inequality for the following verbal sentence: Five plus 2 times a number is less than 20.
2. List the steps you would take to solve the inequality $-5x + 12 < -8$.

Skill Check
Solve the inequality. Graph and check your solution.
3. $4x + 1 > 1$
4. $7 \geq 5x - 3$
5. $\frac{x}{-2} + 6 < -14$
6. $10 > 6 + \frac{y}{5}$
7. $5y + 2 \leq y + 34$
8. $6 + y \geq 2y - 3$

Guided Problem Solving
9. Amusement Parks  You are trying to decide whether to pay $120 for a season pass to an amusement park. If you buy the pass, you get an unlimited number of visits to the park and reduced parking for $8. If you do not buy the pass, you pay $23 admission and $10 for parking each time you visit the park. After how many visits to the park will the cost of visiting with the season pass be less than the cost of visiting without the season pass?

1. Write a variable expression for the cost of making $v$ visits to the park if you don’t buy a season pass.
2. Write an inequality in terms of $v$ showing that the cost of visiting the park with a season pass is less than the cost of visiting the park without a season pass.
3. Solve the inequality.

Practice and Problem Solving

Tell whether the given number is a solution of $5x - 10 > 2x + 4$.
10. 8  
11. 5  
12. 4  
13. -2

Solve the inequality. Graph your solution.
14. $2y + 7 > 11$
15. $6n - 3 \leq -9$
16. $11 - 4z < -1$
17. $3m - 8 > -30 + 5m$
18. $19 \geq \frac{x}{90} - 25$
19. $3 + \frac{b}{3} < 7$
20. $14p - 5 \geq -3p + 114$
21. $-3x - 3 < 2x - 83$

22. Movie Rental  At a video store, you have two options for renting movies. You can pay $4 per movie, or you can pay a one-time membership fee of $10 and then pay only $1.50 per movie. After how many movie rentals will the cost of renting movies with the membership be less than the cost of renting movies without the membership?
23. **Advertising** A small company has an advertising budget of $15,000. The company plans to produce and air a television commercial. It will cost $500 to produce the commercial and an additional $50 each time the commercial is aired. How many days can the company afford to run the commercial if it is aired once a day?

24. **Error Analysis** Describe and correct the error in solving the inequality $4x > 6x + 3$.

Solve the inequality. Graph your solution.

25. $4(5 - 3b) > 4b + 4$

26. $\frac{x - 2}{3} > 4$

27. $3y - 5 < 2(17 - 5y)$

28. $\frac{x + 5}{3} \leq 2$

29. $\frac{-5s - 8}{4} \geq -22$

30. $-3 \leq \frac{2x + 4}{4}$

31. **Fundraising** You are designing greeting cards on your computer to raise money for a charity. You buy card stock at a cost of $.50 per card and rent a table at the fundraiser for $20. You will sell the cards in sets of 12 for $10.20. How many sets of cards do you have to sell in order to make more than what you spend?

32. **Long-Distance Calls** The table gives information about three long-distance telephone companies. For each company, the table gives the monthly fee and the charge per minute for making long-distance calls.

<table>
<thead>
<tr>
<th>Long-Distance Rates by Company</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
</tbody>
</table>

a. After how many minutes of long-distance calls is the cost of using company A for one month less than the cost of using company B for one month?

b. After how many minutes of long-distance calls is the cost of using company C for one month less than the cost of using company B for one month?

c. **Interpret and Apply** If you spend 150 minutes each month making long-distance calls, which company should you use? Explain why.

33. **Fitness** You want to burn at least 300 calories while swimming laps. You burn 12 calories per minute swimming the breaststroke and 9 calories per minute swimming the backstroke. If you have 30 minutes to swim laps, what is the minimum amount of time you should spend swimming the breaststroke?
34. Extended Problem Solving You and a friend join different health clubs. You pay a one-time membership fee of $150 and a monthly fee of $35. Your friend pays a one-time membership fee of $100 and a monthly fee of $40.

a. Analyze Let \( m \) be the number of months that you and your friend have been health club members. Make a table with a column for the number \( m \) of months, a column for the amount you have paid after \( m \) months, and a column for the amount your friend has paid after \( m \) months. Complete the table for whole-number values of \( m \) from 1 to 12 to represent one year of membership at each health club.

b. Make a scatter plot of the data from part (a). Show months on the \( x \)-axis and the amount paid on the \( y \)-axis. Plot points representing the amount you have paid in blue and the amount your friend has paid in red.

c. Writing Using the scatter plot, determine the number of months you and your friend need to be members of your health clubs before you have paid less than your friend. Explain your reasoning.

d. Check your answer to part (c) by writing and solving an inequality.

Mixed Review

Give the coordinates of the point. (Lesson 1.8)

35. \( A \) 36. \( B \) 37. \( C \) 38. \( D \) 39. \( E \) 40. \( F \)

Simplify the expression. (Lesson 2.3)

41. \( 13(2a + 1) \) 42. \( 12 + c + 8 \) 43. \( 5a + a \)

Algebra Basics Solve the equation. Check your solution. (Lesson 3.2)

44. \( 3(x + 4) = 9 \) 45. \( 4(2d + 1) = 28 \) 46. \( -10 = 2(7 - 2x) \)

47. Write and solve an equation for the following verbal sentence: Nine more than 3 times a number is equal to 7 less than twice the number. (Lesson 3.3)

Standardized Test Practice

48. Multiple Choice Which graph shows the solution of the inequality \( 7 - 6x \geq 13? \)

A.  

B.  

C.  

D.  

49. Multiple Choice Which number is a solution of the inequality \(-7x + 3 < -7.5? \)

F. \(-3\) G. \(-1\) H. \(1\) I. \(3\)
Chapter Review

Vocabulary Review

1. Copy and complete: The value of a variable that, when substituted into an inequality, makes a true statement is a(n) ___.

2. Give an example of an inequality.

3. Copy and complete: The inequalities $2x < 2$ and $x < 1$ are ___ inequalities.

4. Are $-2x > 6$ and $x > -3$ equivalent inequalities? Why or why not?

3.1 Solving Two-Step Equations

Example Solve the following problem.

A one-year membership in a video rental club costs $10. Members pay $1.25 per video rental. You spend $45 in one year. How many videos did you rent?

Solution

Let $v$ represent the number of videos you rented. Write a verbal model.

<table>
<thead>
<tr>
<th>Total amount</th>
<th>Club fee</th>
<th>Charge per video rental</th>
<th>Number of video rentals</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>10</td>
<td>1.25v</td>
<td></td>
</tr>
</tbody>
</table>

$45 = 10 + 1.25v$  Substitute.

$45 - 10 = 10 + 1.25v - 10$  Subtract 10 from each side.

$35 = 1.25v$  Simplify.

$\frac{35}{1.25} = \frac{1.25v}{1.25}$  Divide each side by 1.25.

$28 = v$  Simplify.

Answer You rented 28 videos.

5. Spaghetti Your friend bought a box of spaghetti for $1.59 and 2 jars of spaghetti sauce. The total cost was $6.49. Find the cost of one jar of sauce.
### 3.2 Solving Equations Having Like Terms and Parentheses

**Goal**

Solve equations having like terms and parentheses.

**Example**

Solve $2x - x + 1 = 5$ and $4(3r - 9) = 36$.

**a.**

$2x - x + 1 = 5$

Write original equation.

$x + 1 = 5$

Combine like terms.

$x + 1 - 1 = 5 - 1$

Subtract $1$ from each side.

$x = 4$

Simplify.

**b.**

$4(3r - 9) = 36$

Write original equation.

$12r - 36 = 36$

Distributive property

$12r - 36 + 36 = 36 + 36$

Add $36$ to each side.

$12r = 72$

Simplify.

$r = 6$

Simplify.

**Solve the equation. Check your solution.**

6. $17h - 47 + 6h = 160$
7. $2(4p + 8) = 128$
8. $6(w - 4) + 18 = 30$

### 3.3 Solving Equations with Variables on Both Sides

**Goal**

Solve equations with variables on both sides.

**Example**

Solve $13n - 45 = 36 + 4n$.

$13n - 45 = 36 + 4n$

Write original equation.

$13n - 45 - 4n = 36 + 4n - 4n$

Subtract $4n$ from each side.

$9n - 45 = 36$

Simplify.

$9n - 45 + 45 = 36 + 45$

Add $45$ to each side.

$9n = 81$

Simplify.

$n = 9$

Simplify.

**Solve the equation. Check your solution.**

9. $11t + 14 = 95 - 16t$
10. $9n + 64 = -144 - 17n$
11. $3 + 2x = 2(2 + x)$
12. $3(2 + 6b) = 18b$
3.4 Solving Inequalities Using Addition or Subtraction

**Example** Solve \( x + 13 \leq 20 \). Graph your solution.

\[
\begin{align*}
\text{Write original inequality.} \\
x + 13 &\leq 20 \\
x + 13 - 13 &\leq 20 - 13 \\
x &\leq 7 \\
\end{align*}
\]

\( x \leq 7 \)

\[ x \leq 7 \]

\[ -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \]

\( x \leq 7 \)

**Goal** Solve inequalities using addition or subtraction.

13. \( y + 11 < 23 \)  
14. \( 15 \geq z + 9 \)  
15. \( x - 5 \leq 14 \)  
16. \( m - 8 < 26 \)

3.5 Solving Inequalities Using Multiplication or Division

**Example** Solve \( 5x > 30 \) and \( \frac{t}{-8} \leq 5 \). Graph your solutions.

a. \( 5x > 30 \)

\[
\begin{align*}
\text{Write original inequality.} \\
\frac{5x}{5} &> \frac{30}{5} \\
x &> 6 \\
\end{align*}
\]

\( x > 6 \)

b. \( \frac{t}{-8} \leq 5 \)

\[
\begin{align*}
\text{Write original inequality.} \\
-8 \cdot \frac{t}{-8} &\geq -8 \cdot 5 \\
t &\geq -40 \\
\end{align*}
\]

\( t \geq -40 \)

**Goal** Solve inequalities using multiplication and division.

17. \( 3 > \frac{a}{-9} \)
18. \( \frac{b}{7} \geq 13 \)
19. \( 12c \leq 96 \)
20. \( -68 < -17d \)

21. \( -2 > \frac{r}{-6} \)
22. \( 196 \leq 14z \)
23. \( 7h < -56 \)
24. \( \frac{p}{5} > -6 \)
3.6 Solving Multi-Step Inequalities

Goal
Solve multi-step inequalities.

Example
Solve $-8y + 5 \leq 29$ and $3x - 5 > 6x + 13$. Graph your solutions.

a. $-8y + 5 \leq 29$
   Write original inequality.
   $-8y + 5 - 5 \leq 29 - 5$
   Subtract 5 from each side.
   $-8y \leq 24$
   Simplify.
   $\frac{-8y}{-8} \geq \frac{24}{-8}$
   Divide both sides by $-8$.
   $y \geq -3$
   Reverse inequality symbol.
   y ≥ -3
   Simplify.

b. $3x - 5 > 6x + 13$
   Write original inequality.
   $3x - 5 - 3x > 6x + 13 - 3x$
   Subtract 3x from each side.
   $-5 > 3x + 13$
   Simplify.
   $-5 - 13 > 3x + 13 - 13$
   Subtract 13 from each side.
   $-18 > 3x$
   Simplify.
   $\frac{-18}{3} > \frac{3x}{3}$
   Divide each side by 3.
   $-6 > x$
   Simplify.

✓ Solve the inequality. Graph your solution.

25. $-8m - 6 < 10$
26. $8p + 1 \geq 17$
27. $24 \geq 5z - 6$

28. $8 > 2 + \frac{b}{3}$
29. $\frac{p}{28} + 3 \leq 9$
30. $\frac{n}{3} + 4 > 5$

31. $12 - 4q \geq 6q + 2$
32. $6x - 5 > 12x + 1$
33. $6(3 - a) \leq 8a - 10$

34. Snowboarding A ski resort charges $45 for an all-day lift pass and $40 per day for renting boots and a snowboard. At a store, you can buy boots and a snowboard for $360. How many times must you go snowboarding at the ski resort for the cost of buying your own boots and snowboard to be less than renting them?
1. \(7f + 5 = 68\)  
2. \(14 - 3g = 32\)  
3. \(\frac{h}{3} - 14 = -11\)  
4. \(\frac{z}{-2} + 5 = 7\)  
5. \(12 - 2m + 5 = -1\)  
6. \(-6y + 4 + 11y = -16\)  
7. \(3(8 - a) = 12\)  
8. \(-6(3x + 15) = 18\)  
9. \(5t + 5 = 5t - 4\)  
10. \(2n - 6 = -8n + 14\)  
11. \(8b + 4 = 4(b - 7)\)  
12. \(16p + 8 = 2(8p + 4)\)

13. **Movie Tickets** A family of four goes to a movie theater and spends $30.50. They buy 2 tickets for children at $3.50 per ticket, 2 tickets for adults, and 3 boxes of popcorn at $2.50 per box. What is the cost of one adult movie ticket?

14. **Ocean Water** The more salt that ocean water contains, the lower the temperature at which it freezes. Some ocean water freezes at temperatures of \(-1.9^\circ C\) or less. Write and graph an inequality to show the temperatures at which this ocean water freezes.

15. \(x + 75 > -125\)  
16. \(w - 18 < -10\)  
17. \(\frac{t}{12} \geq 3\)  
18. \(-3a - 6 \leq -9\)  
19. \(4(2 - d) \geq -12\)  
20. \(2c - 5 < -21 - 2c\)

21. **School Supplies** You go to the store to buy supplies for class. You want to buy 5 identical folders. The most you can spend is $5.75. What are the individual folder prices that you can afford?

22. Nine is greater than or equal to 15 minus a number.

23. Eight times the sum of 5 and a number is less than 56.

24. Fifteen is greater than 3 times the difference of a number and 4.

25. Seven times a number minus 5 is less than or equal to 16.

26. **Making Bread** A bread-making machine costs $99. The ingredients to make a one pound loaf of bread cost $.45. At a store, you pay $2.19 for the same size loaf of bread. How many whole loaves of bread will you have to make in order for the cost of the machine and ingredients to be less than the cost of buying an equivalent amount of bread at the store?
1. What is the solution of the equation 
   
   \[-4(n + 5) = -32\]  
   A. \(-13\)  B. \(-12\)  C. 3  D. 13

2. Which equation has no solution?  
   F. \(4t - 8 = 4(t - 2)\)  
   G. \(3(r - 1) = -2(2 + r)\)  
   H. \(6p + 2 = 9p - 4\)  
   I. \(7(s + 1) = -3 + 7s\)

3. Giants Stadium in New Jersey can seat up to 80,242 people. Which inequality represents the number \(n\) of people that the stadium can seat?  
   A. \(n < 80,242\)  B. \(n > 80,242\)  
   C. \(n \leq 80,242\)  D. \(n \geq 80,242\)

4. What is the solution of the inequality 
   \[\frac{z}{-4} + 3 < 15\]  
   F. \(z < -48\)  G. \(z > -48\)  
   H. \(z < -3\)  I. \(z > -3\)

5. What is the solution of the inequality 
   \(-12 > y + 6\)  
   A. \(y < -18\)  B. \(y < -6\)  
   C. \(y > -18\)  D. \(y > -6\)

6. Which value is not a solution of the inequality 
   \(-5y - 2 \geq 30.5\)  
   F. \(-162.5\)  G. \(-10\)  
   H. \(-6.5\)  I. \(-3\)

7. Gridded Response What is the solution of the equation \(\frac{t}{5} + 12 = 10\)?

8. Gridded Response The perimeter of the triangle shown is 15 units. What is the value of \(x\)?

9. Short Response Two of your friends go bowling. One friend rents a pair of bowling shoes for $3 and bowls 3 games. The other friend brings his own bowling shoes, bowls 4 games, and buys a soda for $.50. Both friends spend the same amount of money. Show how you can write and solve an equation to find the cost of one game.

10. Extended Response The table below shows the cost of renting a moving van for 1 day from two companies. The daily charge and the charge per mile are given.

<table>
<thead>
<tr>
<th>Company</th>
<th>Daily charge</th>
<th>Charge per mile</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$80</td>
<td>$0.35</td>
</tr>
<tr>
<td>B</td>
<td>$75</td>
<td>$0.39</td>
</tr>
</tbody>
</table>

   a. How many miles \(m\) will you have to drive before the cost of renting a van for one day from company A is less than the cost of renting a van for one day from company B? Express your answer as an inequality.

   b. Graph your inequality from part (a).

   c. Which company is less expensive if you drive 100 miles in one day? Explain.
**Strategies for Answering Gridded Response Questions**

After you solve a gridded response problem, you will have to enter your solution on a grid like the one shown. This involves writing a number in the grid boxes and then filling in the appropriate bubbles below. The grid shown allows you to enter the following types of numbers:

- an integer with up to 6 digits
- a positive or negative decimal with up to 5 digits
- a positive or negative fraction whose numerator and denominator together can have up to 5 digits

**Problem 1**  A roller coaster gave rides to 1780 passengers in one hour, with only 12 empty seats the entire hour. Each train has eight rows of four seats each, as shown. How many trains ran during the hour?

**Solve the Problem**

1. Write a verbal model and then an equation. Let \( x \) represent the number of trains per hour. There are \( 4 \times 8 = 32 \) seats per train, so \( 32x \) represents the maximum possible riders per hour.

<table>
<thead>
<tr>
<th>Actual number of riders</th>
<th>Maximum possible riders</th>
<th>Empty seats</th>
</tr>
</thead>
<tbody>
<tr>
<td>1780</td>
<td>32x</td>
<td>12</td>
</tr>
</tbody>
</table>

\[ 1780 = 32x - 12 \]

2. Solve the equation.

\[ 1780 = 32x - 12 \]
\[ 1792 = 32x \]
\[ 56 = x \]

The roller coaster ran 56 trains during the hour.

**Grid in the Solution**

Remember that the first column of the grid is reserved for the negative sign. Because the solution is a positive two-digit number, use the second and third columns (or the last two columns) to grid in the solution.
Problem 2: Some photo stores offer an adjusted size of photo that better reflects the ratio of pictures taken with a digital camera. An adjusted size for a 4 by 6 photo is shown. How much narrower (in inches) is this than the traditional 4 by 6 photo?

Solve the Problem

1. Write a verbal model and then an equation. Let \( x \) represent the amount (in inches) subtracted from the traditional width.

\[
\text{Adjusted width} = \text{Traditional width} - \text{Amount subtracted}
\]

\[
5.3 = 6 - x
\]

2. Solve the equation.

\[
5.3 = 6 - x
\]

\[
-0.7 = -x
\]

\[
0.7 = x
\]

The adjusted photo is 0.7 inch narrower.

Grid in the Solution

The first grid shows the number 0.7 gridded as .7 with no leading 0. When gridding in a decimal less than 1, it is not necessary to use a leading zero. Also, \( \frac{7}{10} \) would be an acceptable answer, as shown in the second grid.

Checkpoint

Match the problem with its gridded solution.

1. You eat cereal every day for two weeks until the cereal is gone. When you started, the box contained 24.5 cups. What is the average amount (in cups) that you eat each day?

2. You and 9 friends buy both passes listed in the table. You all qualify for the reduced fares. What total amount of money (in dollars) do the ten of you save off the full fare?

<table>
<thead>
<tr>
<th>Day Pass</th>
<th>Full Fare</th>
<th>Reduced Fare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus &amp; Rail</td>
<td>$2.10</td>
<td>$.75</td>
</tr>
<tr>
<td>Shuttle</td>
<td>$.50</td>
<td>$.10</td>
</tr>
</tbody>
</table>

A. \[ .7 \] \[ \text{or} \] \[ \frac{7}{10} \]

B. \[ 1.75 \]
Gridded Response

1. What is the value of the expression \(15 - 14 \div 2 + 5\)?

2. During the month of January, the average high temperature in Montreal, Canada, is \(-6^\circ\)C, and the average low temperature is \(-15^\circ\)C. How many degrees greater is the average high temperature than the average low temperature?

3. Find the perimeter of the square.

\[49 - 4x\]

3x

4. Sylvie has taken 4 exams in her science class. Her mean exam score is 71. She earns 12 points of extra credit, which her teacher lets her apply to one of her exams. What is Sylvie’s new mean exam score?

5. The cost of a child’s ticket for a museum is lowered by $3 from last year. An adult’s ticket still costs $12. A family of 2 adults and 2 children pay $38 to go to the museum this year. What did a child’s ticket cost (in dollars) last year?

6. What is the solution of \(7(x - 2) - 8 = 9x\)?

7. A soccer goalie has made 175 saves so far this season. The school record is 236 saves in a season. There are 6 games left to play. What is the least number of saves the goalie must make in each of the remaining games to break the school record?

8. The solution of the inequality \(ax - 2 < -4\) is \(x < -1\). What is the value of \(a\)?

9. Evaluate the expression \(|x| + 29.6\) when \(x = -13\).

10. You are using a cube-shaped box to mail a package. Find the volume of the box in cubic inches.

11. Point \(A\) is 7 units from zero on a number line. Point \(B\) is 3 units to the left of the opposite of point \(A\). If the coordinate of point \(B\) is positive, what is its value?

12. You had five math tests this semester. Your scores were 89, 97, 93, 86, and 99. Find the mean of your scores.

13. The perimeter of a deck is 84 feet. Find the perimeter of the deck in yards.

14. Find the value of \(x\) (in centimeters) for the given triangle if the perimeter is 30 centimeters.

15. A jar contains 420 beads. You divide the beads equally into 8 smaller jars for your friends to make necklaces. What is the greatest number of beads that can be put in each jar?

16. A beach has a 480 yard wall separating it from a sidewalk. Every 240 feet, there is a break in the wall for access to the beach. How many access points are there along the wall? (Assume that there is no access to the beach at the ends of the wall.)

17. A point on a coordinate plane is 3 units from the \(x\)-axis and 4 units from the \(y\)-axis. The point is in Quadrant II. Identify the lesser of the point’s two coordinates.
Multiple Choice

18. Which point has coordinates (1, −2)?
   A. A   B. B   C. C   D. D

19. Which of the inequalities has the solution whose graph is shown?
   F. $2x + 10 < 16$   G. $-4x - 5 < 7$
   H. $-3x + 8 < -1$   I. $4 - 2x > 10$

Short Response

20. Profit is the difference of income and expenses. The table shows one store’s profit for each of its first 4 months. Find the mean profit. Explain your method.

<table>
<thead>
<tr>
<th>Month</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>March</td>
<td>$-670</td>
</tr>
<tr>
<td>April</td>
<td>$-340</td>
</tr>
<tr>
<td>May</td>
<td>$320</td>
</tr>
<tr>
<td>June</td>
<td>$400</td>
</tr>
</tbody>
</table>

21. You have at most 3 hours to do homework. You spend 45 minutes on math. You want to divide the time remaining equally among 4 other subjects. Write and solve an inequality to find the number of minutes you can spend on each one. Explain your method.

Extended Response

22. You are painting a room that is 16 feet long, 14 feet wide, and 8 feet high. The room has two identical windows and two identical doors. One door leads to a closet 4 feet long, 4 feet wide, and 8 feet high. You do not plan to paint the closet.
   a. The paint you choose is available in both 1 gallon cans that cover about 400 square feet each and 1 quart cans that cover about 100 square feet each. You plan to put 2 coats of paint on each wall, not including the doors or windows. How much paint should you buy?
   b. A 1 gallon can of paint costs $13.90, and a 1 quart can of paint costs $8.90. How much will it cost to put 2 coats of paint on each wall?
   c. You’ve budgeted $40 for paint. Can you afford to paint the inside walls of the closet, excluding the door, with 1 coat of paint?

23. The table shows the annual fee at two gyms and the fee each time members take an aerobics class.
   a. In a year’s time, which gym is less expensive if you plan to take 4 aerobics classes each month?
   b. Write and solve an inequality to determine the number of aerobics classes for which the total cost for 1 year at gym A is less than that at gym B.
   c. How many aerobics classes should you average each month so that the total cost for 1 year at gym B is less than that at gym A?

<table>
<thead>
<tr>
<th>Gym</th>
<th>Annual fee</th>
<th>Class fee</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$540</td>
<td>$3</td>
</tr>
<tr>
<td>B</td>
<td>$360</td>
<td>$5</td>
</tr>
</tbody>
</table>
Cumulative Practice for Chapters 1–3

Multiple Choice In Exercises 1–8, choose the letter of the correct answer.

1. What is the value of $9 - x$ when $x = 5$? (Lesson 1.1)
   - A. 1  
   - B. 4  
   - C. 14  
   - D. 45

2. How can you write $7 \times 7 \times 7$ as a power? (Lesson 1.2)
   - F. $7 \times 3$  
   - G. $3 \times 7$  
   - H. $7^3$  
   - I. $3^7$

3. What is the value of $2(x + y)^2$ when $x = 3$ and $y = 4$? (Lesson 1.3)
   - A. 28  
   - B. 50  
   - C. 98  
   - D. 2401

4. Which list of integers is in order from least to greatest? (Lesson 1.4)
   - F. $-3, -5, -7, -9$  
   - G. $-5, -3, 0, 4$  
   - H. $-2, 4, -5, 9$  
   - I. $-8, 7, -6, 3$

5. What is the value of $-15 + 9$? (Lesson 1.5)
   - A. $-24$  
   - B. $-6$  
   - C. $6$  
   - D. $24$

6. In 24 hours, the temperature went from $-8^\circ C$ to $12^\circ C$. What was the change in temperature? (Lesson 1.6)
   - F. $-20^\circ C$  
   - G. $-4^\circ C$  
   - H. $4^\circ C$  
   - I. $20^\circ C$

7. What is the value of $\frac{x^2}{y}$ when $x = -4$ and $y = -2$? (Lesson 1.7)
   - A. $-8$  
   - B. $\frac{1}{8}$  
   - C. $\frac{1}{8}$  
   - D. 8

8. In which quadrant is the point $(-2, -3)$ located? (Lesson 1.8)
   - F. Quadrant I  
   - G. Quadrant II  
   - H. Quadrant III  
   - I. Quadrant IV

9. Gridded Response Find the volume (in cubic inches) of the trinket box. (Lesson 1.2)

10. Gridded Response The temperature at midnight was $-6^\circ F$. The temperature increased $10^\circ F$ by noon. What was the temperature (in degrees Fahrenheit) at noon? (Lesson 1.5)

11. Short Response You have $500 in a savings account. You make deposits of $30, $125, $10, $20, and $65, and you make withdrawals of $75, $89, $143, $15, and $20. Write a positive integer to represent each deposit and a negative integer to represent each withdrawal. Find the final balance in your savings account. (Lessons 1.5, 1.6)

12. Extended Response The table shows the number of cell phone subscribers (in millions) in the United States from 1996 to 2001. (Lesson 1.8)

<table>
<thead>
<tr>
<th>Years since 1996</th>
<th>Subscribers (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>44</td>
</tr>
<tr>
<td>1</td>
<td>55</td>
</tr>
<tr>
<td>2</td>
<td>69</td>
</tr>
<tr>
<td>3</td>
<td>86</td>
</tr>
<tr>
<td>4</td>
<td>109</td>
</tr>
<tr>
<td>5</td>
<td>128</td>
</tr>
</tbody>
</table>

a. Explain how to make a scatter plot of the data.

b. Make a scatter plot.

c. Does the scatter plot suggest any relationship between the number of years since 1996 and the number of cell phone subscribers? Explain.
Chapter 2

Multiple Choice In Exercises 13–20, choose the letter of the correct answer.

13. Which property is illustrated by\\ \[2x + (y + 1) = (2x + y) + 1\]? (Lesson 2.1)\\ A. Identity property of addition\\ B. Commutative property of addition\\ C. Associative property of addition\\ D. Associative property of multiplication

14. Convert 1.5 miles to feet. (Lesson 2.1)\\ F. \[\frac{1}{7290}\] foot\\ G. \[\frac{1}{7290}\] mile\\ H. 7920 feet\\ I. 7920 miles

15. Which variable expression is equivalent to \[4x - 6\]? (Lesson 2.2)\\ A. \[2(2x - 6)\]\\ B. \[4(x - 6)\]\\ C. \[2(2x + 3)\]\\ D. \[2(2x - 3)\]

16. What is the value of \[x(y - z)\] when \[x = -2.5, y = 4,\] and \[z = 0.1\]? (Lesson 2.2)\\ F. \[-10.25\]\\ G. \[-9.75\]\\ H. 9.75\\ I. 10.25

17. Identify the like terms of the expression \[-6k - 6 + 4 + 4k\]. (Lesson 2.3)\\ A. \[-6k\] and \[-6\]\\ B. \[-6k\] and \[4k\]\\ C. \[6k\] and \[4k\]\\ D. \[4\] and \[4k\]

18. Jay and Maria are painting a large room. Jay has already painted 200 ft\(^2\) and paints 20 ft\(^2\) per minute. Maria has already painted 350 ft\(^2\) and paints 25 ft\(^2\) per minute. They need to paint 3400 ft\(^2\) in total. Which expression represents the area left to paint after \(m\) minutes? (Lesson 2.3)\\ F. \[2850 - 45m\]\\ G. \[2850 - 500m\]\\ H. \[2850 + 45m\]\\ I. \[3950 - 45m\]

19. What is the solution of \[\frac{z}{-12} = 24\]? (Lesson 2.6)\\ A. \[-288\]\\ B. \[-2\]\\ C. \[2\]\\ D. \[288\]

20. You went shopping with $42.60 and came home with $3.33. How much money did you spend? (Lesson 2.7)\\ F. $3.33\\ G. $38.70\\ H. $39.27\\ I. $45.93

21. Gridded Response Whippets are among the fastest running dogs. Suppose a whippet can run at a rate of 52 feet per second for a short period of time. How many seconds would it take the whippet to run 195 feet? (Lesson 2.4)

22. Gridded Response What is the solution of \[x + 11 = 20 - 7\]? (Lesson 2.5)

23. Gridded Response Find the value of \(x\) (in yards) for the given rectangle if the perimeter is 280 yards. (Lesson 2.5)

24. Gridded Response Sam paid $4.41 for some bananas. The bananas cost $.90 per pound. How many pounds of bananas did Sam buy? (Lesson 2.7)

25. Short Response A square has a perimeter of 84 meters. Explain how to write an equation to find the side length of the square. Then find the side length. (Lesson 2.6)

26. Extended Response At store A, a wide-screen TV sells for $1500 after a $250 mail-in rebate. Store A charges $50 for delivery and setup. Store B promises to sell any TV for $75 less than any competitor’s original price and to include free delivery and setup. (Lesson 2.5)\\ a. What is the original price of the TV at store A?\\ b. Which store offers a better deal? Explain your reasoning.
Chapter 3

Multiple Choice In Exercises 27–33, choose the letter of the correct answer.

27. In 3 years, Tom’s age will be 3 times what his age was 3 years ago. How many years old is Tom now? (Lesson 3.2)
   A. 3  B. 6  C. 9  D. 12

28. Which statement about the equation $2(x - 1) = 3x - (x + 2)$ is true? (Lesson 3.3)
   F. The equation has no solution.
   G. The solution is $-0.5$.
   H. The solution is $-1$.
   I. The equation has every number as a solution.

29. At temperatures less than $-458^\circ F$, helium is a solid. Which inequality describes the temperatures $t$ (in degrees Fahrenheit) at which helium is a solid? (Lesson 3.4)
   A. $t > -458$  B. $t < -458$
   C. $t \geq -458$  D. $t \leq -458$

30. You and a friend have $25 to pay for your dinners at a restaurant. Your friend’s dinner costs $13.35. How much can you spend on your dinner? (Lesson 3.4)
   F. Less than $11.65  G. More than $11.65
   H. At most $11.65  I. At least $11.65

31. A roller coaster has a minimum height requirement to ride it. Alison is 5 ft, 3 in. tall and exceeds the requirement by 8 in. Which inequality describes the height requirement in inches? (Lesson 3.5)
   A. $h \geq 48$  B. $h \geq 55$
   C. $h \geq 61$  D. $h \geq 71$

32. What is the solution of $\frac{h}{7} \geq 14$? (Lesson 3.5)
   F. $h \leq -2$  G. $h \geq -2$
   H. $h \leq -98$  I. $h \geq -98$

33. What is the solution of $4 + 6x \geq -8 + 4x$? (Lesson 3.6)
   A. $x \geq -6$  B. $x \leq -6$
   C. $x \geq -1$  D. $x \leq -1$

34. Gridded Response What is the solution of $-2x + 7 = -11$? (Lesson 3.1)

35. Gridded Response You pay $12.99 for a small pizza and two orders of breadsticks. The pizza costs $7.99. What is the cost (in dollars) of one order of breadsticks? (Lesson 3.1)

36. Gridded Response The area of the rectangle is 28 square units. What is the value of $x$? (Lesson 3.2)

37. Short Response Describe the steps you would take to solve the equation $15z - 12 = 3(14 + 3z) - 12$. Then find the solution. (Lesson 3.3)

38. Extended Response Your school is having a fundraising dance. Your costs include $125 for a DJ and $47.50 for decorations. You plan to charge $4.50 for each ticket. (Lesson 3.6)
   a. How many tickets must you sell before you start making a profit?
   b. How many tickets must you sell to make a profit of at least $300?
   c. How would raising the ticket price to $5.00 affect your answers to parts (a) and (b)? Explain.